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INTERRELATIONSHIPS BETWEEN INCOME REDISTRIBUTION  
AND ECONOMIC GROWTH WITH SPECIAL REFERENCE  
TO SRI LANKA

by

Hilarian Marcus Anthony Codippily

A thesis submitted for the  
degree of Doctor of Philosophy in Economics  
to the  
University of Warwick

February 1979

Department of Economics  
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To

Sheila, Deepthi and Shyami

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Errata to thesis of H.Coddipily

The entries in Tables 2.6 and 2.7 on pages 2.44 and 2.45 respectively should be:

<u>Sector</u>	<u>Gini Coefficient</u>	<u>Population Proportion</u>
Urban	0.377	0.1908
Rural	0.340	0.7091
Estate	0.314	0.1001
All Island	0.356	1.000

On page 4.1., line 9, the Gini Coefficient should be 0.36

On page 2.48., line 6, the figure 0.299 should be 0.314

(iii)  
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## ACKNOWLEDGEMENTS

In presenting this thesis, I wish to express my gratitude first of all to my supervisor Alan R. Roe. His comments on each of the draft chapters, particularly in a situation of having to carry out this study as an external candidate over the last four years, and his guidance and supervision during my period of residence at the University of Warwick are gratefully acknowledged.

My sincere thanks are also due to Graham Pyatt of the Development Research Center, World Bank for introducing me into the subject area of Income Distribution and for offering valuable advice at the commencement of this study. I also owe a debt of gratitude to my initial supervisor S.K. Nath of the University of Warwick who gave me valuable advice on the outline of this study and the much needed encouragement to proceed.

The computer facilities afforded by the University of Warwick and the financial assistance to travel to the United Kingdom provided by the Social Science Research Council, United Kingdom are gratefully acknowledged. My residence period at the University of Warwick during 1977 went a long way in improving this study, particularly through the very stimulating

discussions with members of the staff of the Economics Department. In particular I am indebted to Professor A.K. Dixit for his valuable suggestions in developing Chapter 7 of this study. However, I wish to add that the views expressed in this study are mine and that I am alone responsible for any errors that remain.

Finally, I would like to express my sincere thanks to my sister Miss Shiranee Codippily and to Miss Prema Basnayake for typing the initial drafts, to Mrs. Marie Fernando for her patience and care in the tedious task of typing the final draft and to my wife Sheila who helped me in several ways to complete this study.

SUMMARY

The principal aim of this study is to explore the interrelationships that could exist between the processes of growth and redistribution of incomes by means of a formal model representative of Sri Lanka, and to assess quantitatively the effects of alternative policy options available. The model is based on the Chenery-Ahluwalia model for distribution with growth {1974} and retains some of its original features such as a dualistic pattern of production, differential savings rates and linkages amongst major socio-economic groups through employment. But in many respects this study goes beyond theirs by incorporating the government as a separate entity participating in a growth cum redistribution process, the roles of financial institutions, direct taxes, indirect taxes, subsidies and of foreign aid. In contrast to simulation techniques adopted in the Chenery-Ahluwalia approach, the model in this study is developed in terms of a set of simultaneous differential equations. The model is further extended by introducing considerations of incentives to skilled manpower, optimal growth of incomes of the poor over a finite time horizon, and of resource allocation over the major sectors of the economy.

The main conceptual results include the derivation of the Kuznets pattern concerning the behaviour of income inequality as a country develops and the application of optimal control theory to an economic model consisting of three sectors, two control variables and an objective function to be optimised over a finite time horizon. Policy oriented results are also derived, highlighting in particular, the significance of expanding capital for self employment, the desirability of consumption redistribution rather than income redistribution, the limited impact of subsidies, the importance of the modern (private) sector, the role of skilled manpower in development and the optimal allocation of resources over the major sectors of the economy.

DECLARATION

I declare that this thesis is based on original research carried out by me and that the research material has not been published elsewhere nor presented to any other institution or agency. However the concepts of redistribution of consumption and of the 'incentive effect were presented in my M.A. dissertation entitled "Some Aspects of Income Distribution and Economic Growth with special reference to Sri Lanka" submitted to the University of Warwick in 1974, and these have been made use of in Chapter 5 and 6 respectively. Research work of other authors wherever used or referred to have been duly acknowledged.

CHAPTER IINTRODUCTION

"The enhancement of human dignity and the consequent capacity to lead a fuller, freer, more thoroughly human life is the ultimate objective of development" - Robert S. McNamara (Address to the Board of Governors of the World Bank, 1969)

### 1.1 The Nature and Scope of the Study

Economic growth and income redistribution have emerged today as two major objectives of development planning in most developing countries. For, the main preoccupation of development planning today is not only one of bringing about a greater availability of goods and services to a community as a whole but also one of trying to ensure that these goods and services are distributed as equitably as possible amongst the various segments of the community. It is argued that economic growth by itself cannot have much meaning unless the poorest segments of the community are also benefited. On the other hand it is only through economic growth that there will be anything significant to redistribute. Even the very maintenance of an existing system of social services for a growing population necessitates growth of national income.

Whilst suggestions have been made in the recent literature that the two objectives of economic growth and income redistribution are competing objectives, requiring some form of trade-off (see for example

Cline {1972} ) others contend that higher rates of growth need not necessarily generate greater inequality (see Chenery et al. {1974; pp 13-14}). Some of the country studies presented by Chenery et al. {1974; pp. 253-390} do suggest that income redistribution could be successfully combined with economic growth. This area of debate is directly relevant to Sri Lanka. Successive Governments in Sri Lanka have carried out a wide range of social welfare policies, redistributive in character, such as the food subsidy, subsidies in health, education, transport, minimum wage policies and price control of essential items. It has also been argued that agricultural development policies, motivated by import substitution and which are growth oriented in character have contributed towards reduction of income inequality through increased farmer incomes (see Jayawardena in Chenery et al. {1974; p.275}). It is in this context that questions are being posed as to whether or not redistributive policies have had an impact upon growth of national income in Sri Lanka. Specifically, the present climate of thought on development issues, judged from recent policy



statements (for example, the Budget Speech of November, 1977) is committed towards achieving higher rates of economic growth whilst maintaining and further improving social welfare measures. It is therefore relevant to inquire into the nature of interdependencies that could exist between growth and redistributive policies in Sri Lanka.

The principal aim of this study is to explore the interrelationships that could exist between the processes of growth and redistribution of incomes by means of a formal model with special reference to Sri Lanka. Admittedly no model could possibly capture the manifold complexities of reality. In the basic model and its variants developed in this study, we shall take into account only the more important elements connected with growth and redistribution of incomes and shall concentrate on gaining insight into some of the leading issues involved and their policy implications.

The general approach of this study draws its inspiration from the model for distribution with growth presented by Chenery and Ahluwalia (see Chenery et al. {1974; Ch. XI}) and retains a number of its original features which characterize a developing economy. These include the dualistic modes of production i.e. a

modern sector which uses hired labour and a traditional sector based on self employment, concentration in the ownership of capital, differential access of socio-economic groups to employment possibilities, differential savings behaviour, and differential rates of population growth. The basic theme of their model is one of interdependent growth: that is a situation where growth of income of one segment of a community depends upon another through employment linkages. This may be regarded as a major step in the development of an integrated theory of growth and distribution. For, the respective socio-economic groups are not autonomous entities but are connected through employment linkages. Therefore the growth of incomes of one group would depend upon those of other groups. Such interdependencies must necessarily be taken account of in developing a theory of growth with distribution.

In many respects the model developed in this study goes beyond the Chenery - Ahluwalia model, Firstly, the government is represented as a separate entity participating in a growth cum redistribution process. Secondly, the roles of financial institutions, direct taxes, indirect taxes, subsidies and of foreign aid are explicitly introduced into the model. Thirdly, the model is extended to take account of incentive losses

and gains arising from redistributive measures. Another major aim of this study is to incorporate considerations of optimality into a growth cum redistribution process with a view to inquiring into the nature of interrelationships that could exist under conditions of optimality. The central question posed is, how can the welfare of the poor be optimised within a growth cum redistribution process, over a limited time horizon by using policy instruments available to government? An attempt will also be made in the latter part of this study to discuss the implications of growth, redistribution and employment objectives upon individual sectors of the economy.

The methodology used in this study also differs from that used in the Chenery - Ahluwalia study. The dynamic properties of the Chenery Ahluwalia model are examined in terms of simulation techniques rather than by analytical techniques. Although a mention is made that their model can be written in the form of a set of difference equations, yet in anticipation of certain analytical<sup>problems</sup> they had opted to use simulation techniques. In this study we shall neither use simulation techniques nor difference equations. Instead the model will be developed in terms of a set of simultaneous differential equations.

The scope of this study as outlined above also goes

beyond the ground covered by other studies concerning growth and income redistribution in Sri Lanka. For example, the Marga study {1974} on welfare and growth in Sri Lanka traces the historical development of social welfare policies during the post war years, identifies the shifts in emphasis between welfare and growth at different points of time, and discusses some of the areas of conflict. Objectives, priorities and policies relating to development issues are discussed against a sweep of economic, social and political change over a quarter century. But the entire treatment is qualitative in character, as a result of which it is difficult to derive the quantitative effects of the interaction of one element of the economy upon another or to derive a range of alternative paths in a growth cum redistribution process. The study by Codippily {1974} attempts to discuss at a theoretical level the possible trade-off between welfare and growth, advocates redistribution of consumption rather than income and examines incentive losses and gains arising out of redistributive measures but does not develop the model into one of interdependent growth of the Chenery - Ahluwalia type. Although a short chapter is devoted to income distribution in Sri Lanka, a growth cum redistribution model in the Sri Lankan context is not developed. Karunatilake {1975} examines each major welfare measure in detail, evaluates the costs associated with each measure, with a view to assessing the total

impact of welfare measures and advocates a shift to a growth oriented strategy by means of a ceiling on social service expenditure. But the discussions are descriptive in character and cannot be readily incorporated into an analytical framework. Nevertheless these as well as other studies contain valuable insights and we shall come back to these studies later.

## 1.2 Distributive Considerations

Although questions relating to political equality and social equality have been discussed by political and social thinkers throughout the ages, the ideal of economic equality hardly found adequate treatment in classical economics. Economists like Adam Smith, Ricardo, John Stuart Mill and others, with the exception of Pareto were mainly concerned with the distribution of income between the factors of production namely labour and capital, than with the distribution of personal income by size. One explanation for this limitation could be that questions concerning distribution of income by size do involve social and ethical considerations and explicit value judgements, for which one would have to go beyond the territory of economics. Several political and social thinkers have in fact gone beyond the territory of economics and examined questions relating to the historical evolution of societies, economic inequality in society,

reforms desirable etc. It is in such works, as those of Rousseau, Saint-Simon and Karl Marx, that we come across serious discussions of economic inequality. However what they advocated was not complete economic equality (like political equality) but equity in the distribution of incomes. For example, Saint-Simon advocated equity rather than equality as a guideline for distribution: "to each according to his work" (see Paukert {1973; p.98}) and Marx advocated "from each according to his ability and to each according to his needs" at the latter stages of development of a society. (see Sen {1972 ; p.89}). Pareto, a nineteenth century Italian economist was probably the first to carry out empirical examination in depth in regard to the distribution of incomes in several countries. The remarkable regularity he found in the distribution of incomes led to the formulation of Pareto's Law, estimation of the Pareto constant, and to the implication in Samuelson's words (quoted in Paukert {1973 ; p.101}) "that in all places and all times the distribution of incomes remain the same. Neither institutional changes nor egalitarian taxation can alter this fundamental constant of social sciences".

With the development of welfare economics in the early part of this century, distributive considerations were

beginning to be seen from newer perspectives. Pigou, who might be considered to be the founder of welfare economics based his arguments on five main assumptions (see Nath {1973 : pp 13-14}) and derived the proposition.

"it is evident that any transference of income from a relatively rich man to a relatively poor man of similar temperament, since it enables more intense wants to be satisfied at the expense of less intense wants, must increase the aggregate sum of satisfaction".

But the criticism by Robbins and the aftermath led to a fruitless search for a 'more-or-less ethics-free' theory of welfare. As such, welfare economics concentrated on issues that did not involve conflict between different individuals or groups. If it is not possible to make someone better off without making someone else worse off, the situation is said to be Pareto optimal. Thus, in this so called 'optimal' situation, whatever the disparity between the rich and the poor may be, the criterion of Pareto optimality will not advocate any change. Sen {1972 ; pp. 6-7} comments that

"much of modern welfare economics is concerned with with precisely that set of questions which avoid judgements in income distribution altogether ... The concept of Pareto optimality was evolved precisely to cut out the need for distributive judgements ... The almost single minded concern of modern welfare economics with Pareto optimality does not make that engaging branch of study particularly suitable for investigating problems of inequality"

As an attempt to go beyond this impasse, a school of thought developed with the outlook that if questions of income distribution are to be settled, value judgements are inevitable, and that welfare economics should indicate prescriptions for social policy which can lead to decisions. In the words of Nath {1973 p. 13}:

"welfare economists cannot take decisions on behalf of society, but they can, and they do make prescriptions for social policy, which may or may not become decisions".

One of the earliest attempts to go beyond Pareto-optimality was the Bergson-Samuelson social welfare function (SWF) approach.<sup>1</sup> A SWF is a relationship which expresses the social welfare of a society in terms of certain relevant variables. A SWF can even be taken to mean a statement of objectives of a society with some implicit relative weights. The variables may include per capita income, income distribution, employment, environment etc.

Apart from distributive considerations, it becomes evident from the work of Sen {1973 p. 3}. Atkinson {1970} and others, that even in the very measurement of income inequality, that value judgements are inevitable.<sup>2</sup> It would thus appear that

<sup>1</sup> In fact, the phrase "social welfare function" was first introduced by Bergson in 1938.

<sup>2</sup> These measures will be discussed in Chapter 2.



the entire question of income distribution will have to be discussed in terms of value judgements and ethical considerations.

### 1.3 Economic Growth and Income Inequality

Perhaps the first serious attempt to hypothesize on the relationships that could exist between income inequality and economic growth was that of Kuznets' {1955}. He posed the questions "Does inequality in the distribution of income increase or decrease in the course of a country's economic growth? What factors determine the secular level and trends of income inequality?" He advanced the general proposition that as a low income country develops, the extent of income inequality tends to increase at first, then become stable before it begins to decrease. Much later, Paukert {1973} set out to test Kuznets' hypothesis and his conclusions indicated that there was empirical evidence to support Kuznets' hypothesis. He found that there is a tendency for income inequality to increase as countries progress from below the per capita income level of US \$ 100 to the US \$ 101-200 level, the peak of income inequality being reached between the levels of US \$ 200 and US \$ 500.

Since Paukert's study is an inter-country study a question may be posed as to whether a historical trend in a country should necessarily follow a pattern thrown up by a cross section study. However, the work of Soltow in respect of Gt. Britain and Norway points towards a clear long term trend towards equality despite periods of stability or even short term reversals (see Paukert {1973 ; pp. 102, 103, 120}).

More recently, similar results have been obtained by Ahluwalia in "Redistribution with Growth" by Chenery, Ahluwalia et.al. {1974 : Ch. 1, p.17}. He finds that the predicted share of the lowest 40 per cent declines sharply up to per capita income levels of \$ 400 and then flattens out, rising steadily after per capita GNP passes the \$ 1,200 level. In parallel with such a movement, the share of the top 20 per cent increases steadily to reach a peak around the \$ 300 - \$ 400 per capita income level and thereafter to decline gradually. Thus, the superposition of these two movements will give rise to an overall pattern in which income inequality will tend to increase at first up to a level of \$ 400 then flatten out and decrease gradually with higher levels of per capita incomes. In other words, the pattern predicted agrees with the Kuznets' hypothesis.

An interesting point has been made very recently by Lydall {1977} in an attempt to understand the Kuznets'

pattern. He argues that economic growth is primarily a process of adopting better technologies with higher output per man and that if in the early stages of development in a country there are shifts in labour from lower to higher technologies, then there would be tendency for income inequality to widen in the early stages of economic growth and decline in the later stages.

The other important empirical study we should take note of is the one carried out by Adelman and Morris {1971}. Although it might appear at first sight that their results are at variance with the Kuznets hypothesis, it will be argued in Chapter 4 that this is not really the case.

The above studies suggest that there could well be a 'natural' path that a developing country will on the average tend to follow. Even if this were true, it need not preclude a country from attempting to deviate from such a 'path'. Furthermore, the behaviour pattern of a specific country could be entirely different. It follows therefore that if we wish to develop the outline for a redistribution cum growth programme for a specific country, we should look more deeply into the trends in income distribution pattern and economic growth of such country and attempt to study the interrelationships between income distribution and economic growth of the

country. A major aim of this study is to develop the outlines of such a strategy in respect of Sri Lanka.

#### 1.4 Plan of this Study

We shall begin this study with an assessment of the extent of income inequality in the country as a whole as well as in urban, rural and estate areas, based on the latest available information, namely the 1973 Survey of Consumer Finances. The changes in the pattern of distribution of income will then be studied. Thereafter an attempt will be made to analyse these results in terms of recent theoretical developments and methods of analysis. International comparisons will be made and other aspects of economic inequality such as disparities in earning levels of various professions. This assessment will form the contents of Chapter 2.

The aim of Chapter 3 will be to identify and discuss some of the factors that could have contributed towards changes in income distribution. Besides taxes and subsidies many redistributive measures have been given effect to in recent times, such as rural development, agricultural production, subsidised health, education and transport services, minimum wage policy and price control of essential items. Each of these policies will be briefly reviewed so as to complement the results of Chapter 2.

Chapter 4 will be the starting point of the substantive part of this study. The basic model for redistribution with growth together with estimates of parameters which reflect the Sri Lanka context will be developed in Chapter 4. We shall show how the distribution of incomes is determined by the distribution of capital stocks, and wages and profitability parameters noting in particular how the income of the poor are dependent upon the income of the rich and government income through employment linkages. The model will be used in its static form to discuss the relative merits and demerits of nationalisation and of using capital for self employment. Thereafter we shall explore the dynamic properties of the model and examine whether the model is capable of generating the behaviour pattern hypothesised by Kuznets, described briefly in the previous section. The interrelationships between considerations of redistribution and of growth will be discussed in detail within the context of three processes. These are (a) growth without specific redistributive measures, (b) growth with specific redistributive measures and (c) redistribution of consumption. Policy implications will be derived wherever possible.

As mentioned earlier, the model developed in this study goes beyond the Chenery - Ahluwalia model in many respects. Chapter 5 contains three major extensions, namely (a) direct

and indirect taxes and subsidies, (b) the role of financial institutions and (c) foreign aid. This chapter will set out the manner in which the extended model behaves over time with respect to several variants determined by selected values of parameters. In particular we shall examine the impact of subsidies upon the system and assess the results of alternative measures that could raise the incomes of the poor.

Redistributive policies are bound to alter the relative income levels of skilled manpower consisting of scientists, engineers, doctors, economists, planners, accountants, managers, and other professionals and consequently incentive losses are bound to arise. Although the process of trading off leisure for work has been discussed by Codippily {1974}, one major inadequacy was that production functions were not explicitly used to demonstrate analytically the manner in which economic growth could slow down. An attempt will be made in Chapter 6 to incorporate skilled manpower as a factor of production into the model already developed and to discuss in analytical terms the effect of incentive losses.

Another major aim of this study is to inquire into the nature of interrelationships that could exist within a

growth cum distribution process under conditions of optimality, by considering the question of optimising the income of the poor over a limited time horizon. Chapter 7 which deals with this question may be regarded as a logical extension of the work of Hamada{1967} concerning the optimal transfer of income in a growing economy. But he has discussed only the limited case where workers do not save at all and only one policy instrument is available. The main contribution of Chapter 7 will be to incorporate the Hamada approach into the model already developed and discuss the more general case where the recipient class also save, government is treated as an explicit entity in a growth cum redistribution process and where there is more than one policy instrument available to government.

Chapter 8 will make a significant departure from the "aggregative" character of the discussions and attempt to examine the implications of growth, redistributive and employment objectives in relation to individual sectors of the economy. The central theme of this chapter will be that of inquiring into optimal patterns of resource allocation in relation to alternative social objectives.

In Chapter 9 we shall bring together the various policy implications derived in the earlier chapters. An attempt

will be made to synthesize these into a few major policy guidelines which could be of some use in the formulation of national plans and policies.

Concluding remarks will be in Chapter 10.

#### 1.5 A Note on Numbering systems, Abbreviations and Symbols used.

The chapters are numbered serially according to Roman numerals.

The pages in each chapter are given an independent series of numerals prefixed by the order of the chapter. Thus 4 - 3 denotes Chapter 4, page 3. A similar system is adopted for the appendices and accordingly III - 7 would denote Appendix III, page 7. The bibliography would have a separate system of page numbering prefixed by the letter B.

Sections in each chapter are given an independent series of numerals prefixed by the order of the chapter. Thus 4.1 would denote Chapter 4 Section 1. The same system is followed in the numbering of equations, tables, figures and maps. But in the case of the appendices, the table numbering is prefixed by the letter A. For example, Table A III - 2 would denote Appendix III Table 2. Equations in each appendix are given an independent series of arabic numerals.

All references are indicated by the name of author and the year of publication of the work cited within double brackets { }.



Where reference is made to several works by the same author in a given year, the letters a, b, c ... are used to distinguish between the respective works. Footnotes appearing on a given page are assigned an independent series of numerals.

In general abbreviations used in the text are defined wherever they are introduced for the first time. The common ones are listed below.

GDP Gross Domestic Product  
 GNP Gross National Product  
 CFS Survey of Consumer Finances  
 SES Socio - Economic Survey  
 UN United Nations  
 SAM Social Accounting Matrix  
 Rs. Rupees (Sri Lanka)  
 m million  
 Th. Thousand  
 c.i.f. Cost, insurance and freight  
 f.o.b Free on board

All symbols are denoted by English or Greek letters and are defined when introduced for the first time. A vector  $x$  will be denoted by  $\underline{x}$  and its diagonalised form by  $\hat{\underline{x}}$ . Matrices, Hamiltonians and the Laplace transform are denoted by capital letters.

CHAPTER 2INCOME DISTRIBUTION IN SRI LANKA

"The outstanding faults of the economic society in which we live are its failure to provide full employment and its arbitrary and inequitable distribution of wealth and incomes" - J.M. Keynes (General Theory, page 372)

## 2.1 General Features of the Country

The Republic of Sri Lanka, known as Ceylon prior to 1972 is an island situated in the Indian Ocean towards the south-east of the southern extremity of India. The island is separated from the Indian sub-continent by a narrow strip of shallow sea of about 20 miles in breadth. The greatest length of the island, North to South is 270 miles and the greatest breadth is 140 miles. The land area is 25,332 square miles - a little over one fourth size of the United Kingdom.

The south-central part of the country is mountainous and ranges in elevation from about 3000 feet above sea level to 7000 feet, with a few summits reaching beyond the latter limit. The mountainous regions are surrounded by an upland belt, about 1000 to 3000 feet above sea level

while the coastal plain occupying the rest of the country is narrower on the West and South but broadens into a vast tract in the North.

The mean annual temperature in the lowlands ranges between 80°F to 82°F but falls steadily with higher elevations. For example, the mean temperature in Colombo , the country's capital is about 81°F, but at Kandy, 1600 feet above sea level it is 77°F and at Nuwara Eliya, 6200 feet above sea level, it is 60°F. The seasonal variation in the mean monthly temperature is relatively small in many parts of the country.

The annual average rainfall varies from below 40 inches in the driest zones in the North West and South East of the country to over 200 inches at certain south-western slopes of the hills. On the basis of climatic differences, the country may be divided into 3 zones: the wet zone which is the south-western region of the country, the dry zone which mainly consists of the northern and north-western sections of the country and an intermediate zone (see map 2-1).

Sri Lanka's population today is about 14 million. Nearly three fourths of the population live in the wet



and intermediate zones, and the country's capital city of Colombo has a population of the order of 600,000. The major ethnic groups in the country are the Sinhalese (72.0%), Ceylon Tamils (11.2%), the Indian Tamils (9.7%), the Moors (6.7%) and other races such as Burghers, Eurasians, Malays and others accounting for less than 1 per cent. Buddhism is the religion of the majority of the Sinhalese and Hinduism that of the Tamils. The Christians, the majority of whom are Roman Catholic, are of all ethnic groups. Buddhists constitute 67.4 per cent of the population, Hindus 17.6 per cent, Christians 7.8 per cent and Muslims 7.1 per cent.<sup>1</sup>

The rate of growth of population showed a marked increase during the post-war years. The growth rate which was 1.5 per cent during the period 1931-46 increased sharply to 2.8 per cent in the period 1946-53. It continued to be high at 2.7 per cent during the period 1953-63, dropped to 2.3 per cent during the period 1963-71 and dropped further to 1.6 per cent in 1974. The rapid increase in population during the post-war years has been primarily due to the extensive development of health services. Some of the noteworthy results were, (a) the death rate which stood at 20.2 per thousand in 1946 declined to 7.9 per thousand in 1972,<sup>1</sup> (b) infant

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<sup>1</sup> Source: Dept. of Census and Statistics, "Population of Sri Lanka, {1974, p.5.}

mortality fell from 141 per thousand live births in 1946 to 50 per thousand in 1968, and (c) the expectation of life at birth increased from 44 for males and 42 for females in 1946 to 65 and 67 respectively in 1968.<sup>1</sup> The malaria eradication campaign of the 1940s is considered to be one of the special factors behind the sharp fall in the death rate. The birth rates which stood at 36.6 in 1960 declined to 29.4 in 1972, and this decline has been one of the principal factors which reduced the population growth rate during the last few years.

The rate of literacy in Sri Lanka is much above the levels prevailing in most developing countries. Among the males, 85.2 per cent were literate and among the females, 70.7 per cent were literate in 1971. Further in 1971, from the population of persons 15 years and over 30.3 per cent had received an education up to primary level 34.1 per cent beyond primary level, 8.2 per cent up to GCE (Ordinary Level) and 1.5 per cent had passed the GCE (A.L) or higher examinations. The balance 25.9 per cent had no formal schooling.<sup>2</sup> These attainments have resulted from a number of

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<sup>1</sup> See Govt. of Sri Lanka, Five Year Plan 1972-76, p. 113.

<sup>2</sup> Source: Dept. of Census and Statistics, Op.cit.p.53.

measures adopted in the educational field, such as the introduction of Free Education, adoption of the mother tongue as the medium of instruction and the overall expansion of educational facilities.

## 2.2 The Economy

From the ancient times<sup>1</sup>, agriculture was the mainstay of the Sri Lankan economy. During the reign of a long series of Sinhalese Kings, a great deal of attention was paid to the development of a vast network of tanks (reservoirs) and irrigation works in order to support domestic agriculture, particularly the cultivation of paddy. The country reached a high level of prosperity particularly in the twelfth century during the reign of King Parakrama Bahu I, who is considered to have been perhaps the greatest builder of tanks, irrigation networks and dagobas in ancient times. Historical records show that the country had sizeable surpluses of rice for export after meeting the requirements of a large population. Economic activity was however interrupted from time to time during the earlier periods, by foreign invasions, particularly from the Indian sub-continent.

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<sup>1</sup> The Mahawansa records the history of the country from 543 B.C., when an Aryan prince by the name of Vijaya arrived with about 700 men from Northern India.

The country had greatly declined in prosperity, mainly through internal divisions and rivalries, at the time of the Portugese arrival in 1505. The economic interests of the Portugese who had settled in certain maritime parts of the island were those of growing cinnamon and other spices, of shipping these to Lisbon and of assuming general control over the spice trade. The Portugese were expelled by the Dutch in the middle of the 17th century. Having settled down in certain parts of the island, the Dutch too assumed control over the spice trade. The Dutch in turn were expelled by the British in 1776. With the fall of the Kingdom of Kandy in 1815 and the Proclamation in 1818, the entire country came under British rule.

The early part of the British period saw the beginnings of a dual economy. Tea, which replaced coffee in the 1870s and rubber and Coconut plantations which were export oriented were steadily developed. Although paddy cultivation and other forms of subsistence agriculture continued to co-exist, their decline was inevitable. The repair of tanks and irrigation works carried out from time to time, by itself, failed to produce significant results. Thus a country which had once exported large quantities of rice began to import rice. Meanwhile the



expansion of tea, rubber and coconut plantations brought about steady increases in export earnings. Tea plantations were pioneered by the British Companies and they controlled the largest number of the bigger estates, which produced the country's best tea. The estates were efficiently managed through a number of Agency Houses, and were in a position to provide attractive dividends. The labour for these estates was obtained from South India since local workers were unwilling to work in plantations. In order to support the plantations sector, large scale development of infrastructure and services were also initiated by the British. Most noteworthy amongst these were the network of roads, railways, postal and telecommunication systems, transport, manufacturing activities for processing of plantation products, the spread of English education and the setting up of import-export trading establishments. These, together with the development of a colonial administration led to the formation of a modern sector. The overall pattern that emerged was the classic dual economy with a large traditional rural subsistence economy side by side with a small but powerful modern sector.<sup>1</sup> The impact of the plantation sector was so great that as much as 98

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<sup>1</sup> A fuller discussion of the pre-Independence period is provided by H.N.S. Karunatilake {1971: pp.1-26}

per cent of the export earnings came from tea, rubber and coconut at the time of Independence in 1948.<sup>1</sup>

The post-Independence period saw a concerted effort to make the country less dependent on imports for her requirements of food and other consumer items. The ancient tanks and irrigation works were restored, large scale irrigation works such as the Gal Oya Project were initiated, vast acreages of jungle land were cleared, a large number of colonists were settled in the various colonization schemes and a range of supporting services such as subsidies on fertilizer and agricultural implements, agricultural credit, guaranteed price schemes, agricultural research,<sup>and</sup> agricultural extension services were introduced. These efforts resulted in significant increases in paddy production. For example, paddy production rose from 31.3 million bushels in 1957 to 70.0 million bushels in 1970.

The process of industrialization which had already commenced during the pre-Independence period received a great stimulus. State Industrial Corporations were set

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<sup>1</sup> Karunatilake {1971; p. 14}

up during the period 1944-1956 for the manufacture of plywood, cement, leather products, oils and fats, paper, caustic soda, chlorine and ilmenite. With the formation of a government in 1956 committed to the principles of democratic socialism, plans were formulated for the further development of state industry. The Ten Year Plan 1959-68, the country's first comprehensive development plan laid heavy emphasis on the need to create a sizeable industrial sector. Amongst the many industries that were set up or developed during the post-1956 period, were the steel, hardware, tyre and flour milling plants installed with assistance from socialist countries. The early 1960s saw a steady deterioration in the terms of trade. Consequently, import restrictions were placed on a wide range of consumer items, which in turn provided an opportunity for setting up local industry. This situation together with the governments policy for supporting industrial ventures through a number of tax incentives gave rise to a significant growth of import substituting private sector industries. The other major thrust of economic activity during the post-Independence period was the development of large scale hydro-power projects, with a view to meeting the requirements of industry.

The system of education which had hitherto produced

clerks and civil servants for a colonial administration also underwent a radical change. Reforms were introduced so as to produce a greater number of scientists, engineers, doctors, accountants and other professional personnel. In schools, the medium of instruction was changed from English to the national languages. The post-Independence period also saw the revival of traditional arts and cultural and religious practices. From the point of view of social welfare, significant developments during the post-Independence period were a system of free education up to University level and subsidies on food and health services. The nature and the implications of these measures will be discussed later.

Agriculture is still the dominant sector of the Sri Lankan economy today, employing over half the working population. The share of Agriculture, Forestry, Hunting and Fishing in the GNP at constant factor cost prices stood at 32.0 per cent in 1977. The other major sectors are Manufacturing (12.6%), Wholesale and Retail Trade (13.6%) and Services (14.3%).<sup>1</sup> The dominance of Agriculture is even more explicit in the case of export earnings. In the composition

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<sup>1</sup> See Appendix I

of total earnings from domestic exports during 1977, Tea accounted for 53 per cent, Rubber for 14 per cent, coconut products for 5 per cent, their joint contribution coming to as much as 72 per cent. These three traditional exports had in fact accounted for larger shares in earlier years. As noted previously, they had accounted for 98 per cent of the export earnings in 1948. In more recent years as for example in 1971, they accounted for 89 per cent of export earnings and for 88 per cent in 1972. The decline in their share in recent years is chiefly due to the expansion in the export of other products such as precious stones, bakery products, fruit juices, garments, leather products, naptha, and marine bunkering.

Throughout the 1960s export earnings showed a steady decline. In fact, per capita exports declined from a level of US \$ 38 in 1960 to US \$ 26 in 1969.<sup>1</sup> The decline is mainly attributable to the steady fall in price of tea and to some extent due to the unfavourable price trends for rubber and coconut. This trend when compounded with rising import prices resulted in a steady deterioration of the terms of trade from 148 in 1960 to 62 in 1976.<sup>2</sup>

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<sup>1</sup>Gamani Corea {1971: p.. 24}

<sup>2</sup>Central Bank of Ceylon, Annual Report 1977, Table 16.

Export earnings have however shown a notable increase over the last few years: total earnings rose from Rs.2,617 million in 1973 to Rs.3,933 million in 1975 and to Rs.6,638 in 1977.<sup>1</sup> This increase came largely from favourable prices for most commodities and to some extent from the increase in volume of non traditional products. However, the increase in export earnings was more than negated by a much higher rise in the import bill. The value of imports increased sharply from a level of Rs.2,715 million in 1973 to Rs.5,251 million in 1975 attributable entirely to increases in import prices. As a result a large balance of payments deficit of Rs.1,318 million was recorded in 1975. However, after a long series of deficits, the balance of payments improved strongly in 1976 and by 1977 a surplus of Rs.1,259 in the current account was recorded. This recovery was principally due to the rise in tea export earnings.

Large balance of payments deficits in recent years resulted mainly from efforts to keep certain minimum levels of imports for consumption and for meeting the requirements of crude oil, fertilizer and industrial raw material, without which economic activity would have

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<sup>1</sup>See Appendix II

drastically declined. Successive deficits have been met mainly from drawings from the I.M.F., Oil Facility, short term credits and suppliers credits. These measures have no doubt placed a heavy debt servicing problem upon the country. The debt service ratio which stood at 23.0 per cent in 1973 declined to 17.8 per cent in 1974 but rose to 22.9 per cent in 1975. However, it declined to 20.1 per cent by 1976 and to 18.5 per cent by 1977.

Since Food and Drink account for a high proportion of import bill<sup>1</sup>, it is clear that much of the efforts in improving the balance of payments position would lie in stepping up domestic food production. At present, the government's development efforts are directed towards this end. Substantial gains are expected from the largest ever irrigation cum power project undertaken by the country namely the Mahaweli Project. The diversion of Mahaweli waters to a number of large tanks in the dry zone and the additional irrigation facilities obtained therefrom will enable a large number of farmers to grow two crops per year instead of one.

As in the case of many developing countries, unemployment has remained a major problem for the past two decades or

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<sup>1</sup> In 1977, Food and Drink had accounted for 36 per cent of the import Bill.

so. The population "explosion" in the 1940s and 1950s referred to earlier led to an upsurge in the youth population in the 1960s. With the growth of the economy being inadequate to keep pace with the entry of youth into the labour market, a serious unemployment problem became inevitable.<sup>1</sup>

One of the foremost objectives of the Ten Year Plan as well as subsequent development plans was that of reducing the levels of unemployment and under-employment and of providing employment for increases in the work force. But employment creation was impeded on the one hand by constraints on capital investment and on the other hand by the shortage of foreign exchange for import of raw material and essential plant and machinery. Investible resources were constrained to a great extent by large outlays on welfare expenditure, mainly in the form of subsidies on food, education and health services.<sup>2</sup> Further, in the

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<sup>1</sup> According to the Labour Force Participation Survey carried out by the Central Bank in 1973, 17.4 per cent of the labour force was unemployed. As much as 77 per cent of the unemployed were in the rural sector.

<sup>2</sup> The magnitude of welfare expenditure by government was such that over the last two decades it not only ranged between 35-45 per cent of recurrent expenditure, but it also exceeded capital expenditure (see Karunatilake, {1975; Table 1, p.23})



context of a scarcity of capital and shortage of foreign exchange, there was no concerted effort to carry through development programmes based on labour intensive technology and local resources on a sufficiently large scale. It was only much later, in the 1970s that the importance of a development strategy based on labour intensive technology and local raw materials began to be recognised. This approach received much emphasis in the Five Year Plan (1972-76) and specific areas of industry were clearly identified. These included textiles, wood products, mining and quarrying, light engineering, paper products and structural clay products, one of the noteworthy programmes initiated by government was the Divisional Development Council Programme (DDC programme). In contrast to capital-labour ratios ranging from Rs.25,000 to Rs.150,000 in large scale capital intensive projects, the capital-labour ratios were as low as Rs.2,000 on the average in the case of the DDC projects. Since these projects were based on local raw materials and locally fabricated machinery to a great extent, the gestation periods were as low as 2 - 3 months. As at the end of 1976, nearly 2,000 such small scale labour intensive projects, set up under a co-operative form of ownership were in operation, providing employment to over 31,000 persons.<sup>1</sup>

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<sup>1</sup> See Gunasekera H.A. de S and Codippily H.M.A {1977} for a further discussion.

Although the Five Year Plan (1972-76) had envisaged a growth rate of 6 per cent as the minimum desirable rate of growth, the actual growth rate achieved during the period 1972-75 was of the order of 3.4 per cent. The reasons for the short fall were more than one. Firstly, droughts in three successive years had devastating effects on the paddy production and on other crops. Secondly, the increase in the price of crude oil had a serious effect upon the economy in more than one way. The price increase of oil added further burdens to the import bill. Further, the high price of oil raised the price of fertilizer to a level beyond the reach of most farmers and the import price of manufactured articles registered a sharp increase.

The Plan period 1972-76 also saw a number of other developments in the economy. From the point of view of improving the balance of payments situation the most noteworthy amongst these were the phenomenal growth of the mining and export of gems (precious stones), rapid expansion of tourism, the building up of a fleet of merchant ships, and an expansion of marine bunkering activities, all of which brought in substantial foreign exchange gains. Reforms were also introduced to bring about <sup>a</sup> more egalitarian form of asset ownership. The most important amongst these were the introduction of ceilings on the ownership of housing property, land reform and the nationalization of foreign

owned estates.

### 2.3 Income Distribution: a Preliminary Description

The principal sources of information regarding income distribution in Sri Lanka are the periodic sample surveys conducted by the Central Bank of Ceylon and the Department of Census and Statistics. In this study, we shall use data from the three Surveys of Consumer Finances (CFS) conducted by the Central Bank in the years 1953, 1963 and 1973, and the Socio Economic Survey (SES) conducted by the Department of Census and Statistics in the year 1969/70.

The CFS 1973 was based on a stratified two stage sample design and the survey sample comprised 28,587 persons in 5088 households. The average household consisted of 5.62 persons. Amongst other information, the survey recorded data relating to income receivers' income for the two month and six month periods immediately preceding the first day of the interview of each income receiver. The two month and six month reference period was not the same all over the island as the field work of the survey had been staggered over a period of two months starting on 3rd January, 1973. As mentioned in their report, the staggering of the reference period would have caught up to some extent the monthly variation in expenditure.

Data on income comprised 'money income' as well as 'income in kind'. Money income included cash receipts as well as the value of produce such as rice and other major cereals. Income in kind formed the imputed value of goods and services enjoyed as part payment in employment, gifts, and other transfer payments together with the imputed value of own garden produce consumed at home. Some of the items that contributed to income in kind were the free rice ration distributed by the government, meals, uniforms, railway warrants, free accommodation enjoyed by employees and home consumed own garden produce and animal husbandry products. Income in kind was valued at what could be ascertained as the market price of these goods and services in the immediate vicinity. However the value of other free government facilities such as educational and medical facilities could not be estimated objectively and were therefore excluded from income in the CFS 1973 ( see p. 53).

Let us briefly note the other concepts and definitions used in the CFS.

Household: A household is a person living alone or a group of persons living together in a housing unit and having common cooking arrangements. The members of a household need not be blood

relations: a household can include boarders and servants. In cases where the number of boarders exceeded three, the household was considered to be running a commercial boarding house and all boarders were accordingly excluded from the household.

Spending Unit: Within households, there are smaller groups which act as more or less independent units for spending purposes. For instance, the families of two brothers can form a household, but one brother's spending can be independent of the other's. A spending unit is defined to consist of one or more persons who are members of the same household, and share major items of expenditure. Servants and boarders will form separate spending units. A person dependent on more than one spending unit within the household was included in the spending unit on which he/she was most dependent.

Income Receiver: A person who has received an income during the six months immediately prior to the survey was referred to as an income receiver.

The definitions of a household, spending unit, income receiver used in the CFS 1963 were essentially the same as above.

The CFS 1973 revealed that the number of spending units per household increased with size of household until the 13 member household was reached and declined thereafter. The average number of income receivers per household showed an increasing trend with the size of household.

Some of the general results in regard to households that emerged from the 1973 CFS were as follows:-

- (i) The average size of a household was 5.63;
- (ii) The mode in the distribution of household size was 4;
- (iii) The average number of spending unit per household was 1.05;
- (iv) The average number of income receivers per household was 1.44;
- (v) The average number of dependents per household was 4.18.

The CFS 1973 presents income distribution both in respect of income receivers as well as in respect of spending units.

In this study we shall be mainly concerned about the

welfare of households and therefore, in the first instance examine the results relating to spending units - a closer entity to households than income receivers. Income distribution with respect to income receivers will also be discussed in the latter part of this chapter.

According to the CFS 1973, the mean monthly income of a spending unit was estimated to be Rs.311 and the median monthly income to be Rs.250. The pattern of income distribution was as shown in Table 2.1 below.

Table 2.1

Distribution of Spending Units by Income Groups  
All Island (1973)

Income Group (Monthly Income)	Percentage of spending Units	Percentage of Two month Income	Percentage of Six month Income
Less than Rs.100	6.90	1.64	1.70
Rs.101 - Rs.200	27.33	13.67	13.61
Rs.201 - Rs.400	45.48	41.33	41.21
Rs.401 - Rs.600	12.75	19.65	19.86
Rs.601 - Rs.800	3.90	8.58	8.70
Rs.801 - Rs.1000	1.64	4.66	4.63
Rs.1000 - and over <sup>1</sup>	2.00	10.47	10.29
Total	100.00	100.00	100.00

Source: Derived from Table 43, Central Bank  
CFS (1973) Part I.

<sup>1</sup> Includes a wide range of incomes such as those of managers in the public sector or in the private sector, self-employed professionals and private entrepreneurs.

It would appear from the above table that nearly 80 per cent of the spending units in Sri Lanka receive incomes of Rs.400 per month or less. Only 3.64 per cent of the spending units receive incomes of Rs.800 per month and over. Those in the group of over Rs.1,000 per month constitute 2 per cent of the Spending Units, but receive over 10 per cent of total income. One cannot immediately comment on what exactly these income levels mean in terms of a standard of living or more specifically in terms of a basket of goods. For, a given income level, say for example Rs.400 per month can reflect widely differing standards of living in terms <sup>of</sup> urban, rural and estate life. Such differences are due to differences in purchasing power of a rupee as between urban, rural and estate sectors. But before discussing such questions, let us get back to the overall pattern <sup>of</sup> income distribution in the country as a whole and inquire whether there have been any significant changes over time. This cannot be done in terms of tables such as 2.1 on account of changes in the purchasing power of the rupee over time. Instead, spending units have to be ranked and changes in the respective shares have to be examined. The CFS 1973 has already done this and the results obtained are reproduced below.



Table 2.2

Percentage of Total Income Received by each  
Ten per cent of Ranked Spending Units

Decile of spending Units	Share of Total Income Received		
	in 1953	in 1963	in 1973
Lowest	1.90	1.50	2.79
Second	3.30	3.95	4.38
Third	4.10	4.00	5.60
Fourth	5.20	5.21	6.52
Fifth	6.40	6.27	7.45
Sixth	6.90	7.54	8.75
Seventh	8.30	9.00	9.91
Eight	10.10	11.22	11.65
Ninth	13.20	15.54	14.92
Highest	40.60	36.77	28.03
Total	100.00	100.00	100.00

Source: Central Bank, CFS (1973)  
Part I Table 45 (p.60)

The most significant features in the above table are:

- (a) the marked decline in the share of income of the highest decile from a level of 40.60 per cent in 1953 to 28.03 per cent in 1973, and
- (b) the improvement in the share of income of the lowest decile from levels of 1.90 per cent in 1953 and 1.50 in 1963 to 2.79 in 1973.

Improvements are also seen in the case of all other deciles, with more accentuated increases being evident in the lower deciles. These improvements are considered to have come about partly as a result of conscious redistributive policies and partly as a result of certain import substitution programmes, particularly the food production programmes carried out by successive governments over the last two decades or so. The more important aspects of these policies and programmes will be discussed in Chapter 3.

It would appear from Table 2.2 that notable progress has been made in Sri Lanka in achieving a more equitable distribution of income in a relatively short period of time. A point of immediate interest is to inquire as to how the results achieved compare with those of other countries. A broad international comparison is given in Table 2.3.

The noteworthy features in Table 2.3 are that the bottom forty per cent of the households in Sri Lanka enjoyed a relatively high share of total income among countries listed in the table and that the top twenty per cent of the households received a relative share of income less than those prevailing in the other countries. An element of caution is however necessary since the distribution within the group of poorest households are not brought out in such

broad comparisons.

Table 2.3

An Inter-country Comparison of Income Distribution

(A selection of Developing Countries based on  
availability of recent information)

Country	Percentage of Total Income received by the			
		Lowest 40%	Middle 40%	Top 20%
		of households		
Argentina	(1970)	16.5	36.1	47.4
Brazil	(1970)	10.0	28.4	61.5
India	(1964/65)	18.6	33.9	47.5
Ivory Coast	(1970)	10.8	32.1	57.1
Korea	(1970)	18.0	37.0	45.0
Malaysia	(1970)	11.6	32.4	56.0
Peru	(1971)	6.5	33.5	60.0
Philippines	(1971)	11.6	34.6	53.8
Sri Lanka	(1973)	19.2	37.8	43.0
	(1969/70)	17.5	39.4	43.1
Thailand	(1970)	17.0	37.5	45.5
Uganda	(1970)	17.1	35.8	47.1

Sources: "Redistribution with Growth" (1974)  
Chenery, H., Ahluwalia, M.S., et al.

Central Bank of Ceylon Survey of Sri  
Lanka's Consumer Finances 1973" Part I.

Department of Census and Statistics,  
Socio Economic Survey (1969/70).  
Pranab, K. Bardhan, "The Pattern of  
Income Distribution in India, A Review"  
In Poverty and Income Distribution in  
India ed. T.N. Srinivasan and P.K.  
Bardhan.

For purposes of broad comparison it is also interesting to note that the average income share of the lowest forty per cent in all developing countries taken as a group is estimated at only 12.5 per cent and the corresponding figure for developed capitalist countries is estimated at 16 per cent. Whilst both estimates are exceeded by the Sri Lanka figure, it should however be noted that the latter falls short of the figure for socialist countries estimated at about 25 per cent. The low degree of income inequality in socialist countries is largely attributed to the fact that income from ownership of capital does not accrue as income to individuals. (see Chenery, Ahluwalia et al {1974 :p.7}).

We have discussed so far only some aspects of the pattern of income distribution in the country as a whole. In order to proceed with the inquiry into the more detailed aspects of income inequality, one must necessarily examine the subject in terms of type of household and by geographical regions. Households or spending units may be divided into three types namely by their location in urban, rural and estate sectors. These types are defined as follows:-

Urban - Households/Spending units in Municipal,  
Urban and Town Council areas.

Estate - Households/Spending units in tea and  
rubber estates of over 20 acres and  
with more than 10 resident workers.

Rural - Households/Spending units not classified as urban or estate.

To proceed with the discussion, two aspects ought to be examined separately:

- (a) the distribution of total income between sectors;
- (b) the distribution of income within each sector.

Inequalities in income distribution between sectors could be seen both in terms of mean incomes as well as in terms of shares of total incomes received. Results of computations made from Tables 580-583 of the CFS are given in the table below:-

Table 2.4  
Mean Income and Shares of Income by Sector - 1973

Sector	Mean Income per Sp. Unit per month	Percentage of Spending Units	Percentage of total 2 month income
Urban	Rs. 397	19.08	24.39
Rural	291	70.91	66.41
Estate	285	10.01	9.20
All Island	311	100.00	100.00

It would appear from the above table that the mean income of an urban spending unit is 36 per cent higher than that of a rural spending unit. But this is a comparison made purely in money terms and could grossly understate rural income in real terms. For, rural prices of several essential commodities, chiefly food items, are significantly lower than urban prices, and therefore one would have to make an adjustment for the

difference in price levels, in order to make a meaningful comparison. This could be done by the use of a price index which could be constructed in terms of a comparable basket of goods and a set of reliable prices of commodities in urban and rural areas. But the absence of a reliable set of prices of a representative character in urban and rural areas precludes such a comparison. On the other hand one could get a general idea of the disparities in real income as between urban, rural and estate households by comparing the actual basket of goods an average household is able to purchase in each case. Such a comparison is attempted in Appendix III and the main results are summarised below:

The general conclusion that emerges in regard to food items consumed per head is that except in the case of meat, fish, eggs and milk, there do not seem to be marked disparities in consumption per capita as between urban, rural and estate households. This disparity is perhaps partly offset by the higher consumption of vegetables and fruit in rural areas and high consumption of protein rich pulses in estate areas.

In the case of clothing and footwear, Table A III - 2 shows that the average expenditure by an estate spending unit is comparable with that of an urban spending unit. A possible explanation is that estate areas are mainly located in the

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hill country where the climate is relatively cold; this necessitates greater expenditure by estate households. But the average expenditure per rural spending unit is significantly below that of an urban spending unit. However, one cannot immediately conclude that the rural spending unit is worse off in regard to clothing since neither climatic conditions in most areas nor the rural life styles nor rural occupations such as paddy cultivation necessitate high expenditure on clothing.

As regards housing, the fairly obvious result that emerges from the comparative study (see Table A III - 3) is the relatively unsatisfactory nature of estate housing conditions. Due to wide variations in circumstances it is difficult to make any other meaningful comparisons.

Two other disparities emerge from the comparative study. Firstly, the expenditure on and the ownership of durable consumer goods are relatively higher in urban areas. Secondly, expenditure on education by spending units in estate areas is found to be significantly below those of their urban and rural counterparts.

The comparative study in Appendix III also shows that economic changes have distinctly moved in favour of the non-urban community during the period 1963 to 1973. It is estimated .

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that the non-urban population (80.9% of total) had received 71.7 per cent of total income in 1963, whereas by 1973 the non-urban population (77.6%) had received 75.6 per cent of total income. Real income per urban spending unit had declined by 1.5 per cent per annum during this period whereas real income per rural and estate spending units had increased at rates of 1.7 and 0.9 per cent per annum over the same period.

Let us now examine the second aspect we set out to discuss, namely, the distribution of income within each sector. The shares of income received by each quintile of ranked spending units in the urban rural and estate sectors for the years 1963 and 1973 are shown in Table A III - 7 of Appendix III. This table clearly shows a marked improvement in the shares of income accruing to the lowest quintiles both in the urban and rural areas. A significant decline in the share of income received by the top quintile both in the urban and rural areas is also seen. The results in this table are further summarised in the table below, to show how the shares of income of the lowest 40 per cent, middle 40 per cent and top 20 per cent have changed in the respective sectors.



Table 2.5

Percentages of income received by the bottom 40 per cent, middle 40 per cent and top 20 per cent in Urban, Rural and Estate Sectors (1963 and 1973)

Class of Spending Unit	Urban		Rural		Estate	
.	Share of total income received					
	1963	1973	1963	1973	1963	1973
	10.7	17.4	14.3	20.0	22.4	21.6
	32.1	37.4	36.2	38.5	38.8	37.9
	57.2	45.2	49.5	41.5	38.8	40.5
	100.0	100.0	100.0	100.0	100.0	100.0

Source: Derived from Table A III-7

The results in the above table indicate that during the period 1963-73 income inequality had reduced only in the urban and rural sectors; it is perhaps mainly in these terms that one could explain the overall reduction in income inequality<sup>1</sup> in the country as a whole during the period 1963-73 (see Table 2.2).

The analysis in Appendix III also shows that total real income had increased by 51.1 per cent over the period 1963-73 (see Tables A III - 9 and A III - 10). Since the relative shares of income of the lower income groups in the urban and rural sectors had improved over the same period, it would follow that the increase in total income has been distributed in favour of the

<sup>1</sup> In general population shifts also affect income inequality

lower income groups in urban and rural areas. A little less than a third of the total increase in real income had reached the two lowest quintiles in the urban, rural and estate sectors.

#### 2.4 Income Distribution: some Numerical Summaries

A wide range of measures of income inequality has been recently discussed in the literature. These measures fall into two categories, namely, positive measures and normative measures. The aim in using a positive measure is to assess objectively the extent of income inequality by employing a purely statistical measure of relative variation of income<sup>1</sup>. On the other hand normative measures are based on explicit concepts of social welfare. But as argued by Sen {1973 ; p.3}, the distinction between them is not so clear cut. Our interest in measuring income inequality is related to our normative concern with it. Conversely, normative considerations are inevitable in judging the relative merits of various measures of income inequality. As shown later, a normative interpretation could be given to the so called 'positive' measures. It should also be noted that a normative measure may not capture the totality of our ethical evaluation. In empirical work, as in the case of this study, it is perhaps convenient to use positive measures of income inequality. But since welfare considerations are an integral part of this study we shall also use one normative measure. As noted earlier, if questions relating to income distribution are to

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<sup>1</sup> See Sen {1973 ; p.2}

be discussed, value judgements based on some concepts of social welfare are inevitable.

Amongst a number of measures of inequality that have been widely discussed in the literature and used extensively in empirical work, the more important ones are:-

(1) The Coefficient of Variation

which is defined by -

$$C = \sqrt{\frac{\sum (y_i - \mu)^2}{n}} / \mu \quad \text{----- (2.1)}$$

where  $y_i$  = income of individual  $i$

$n$  = number of individuals

$\mu$  = mean income

(2) The Standard Deviation of Logarithms

defined by -

$$H = \left[ \sum (\log \mu - \log y_i)^2 / n \right]^{1/2} \quad \text{----- (2.2)}$$

where  $y_i, \mu$ , and  $n$  are defined as in (2.1) above. (See Sen {1973; p. 29})

(3) The Gini Coefficient

which may be defined by -

$$G = \frac{1}{2\mu n^2} \sum_{i=1}^n \sum_{j=1}^n |y_i - y_j| \quad (2.3)$$

the notations being the same as in (2.1)

(4) The Atkinson Index

which may be defined by -

$$I = 1 - \left[ \sum_{i=1}^n \left( \frac{y_i}{\mu} \right)^{1-\epsilon} f(y_i) \right]^{\frac{1}{1-\epsilon}} \quad (2.4)$$

where  $y_i$  and  $\mu$  retain the same definition and

$\epsilon$  denotes the "degree of inequality aversion",  
and  $f(y_i)$  = proportion of population with income  $y_i$ .

(5) Sen's Measure of Poverty

defined by -

$$P = \frac{2}{y^* n^2} \sum_{i=1}^n (y^* - y_i)(q - i + \frac{1}{2}) \quad (2.5)$$

where  $y^*$  represents the poverty line,  
 $y_i$  are arranged monotonically, and  
 $q$  = proportion of population below  $y^*$ .

The somewhat lesser known and perhaps the less important measures of inequality are:-

(6) The proportion by which the Geometric Mean falls short of the Arithmetic Mean,

(7) The proportion by which the Harmonic Mean falls short of the Arithmetic Mean,

(8) The Pareto constant

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<sup>1</sup>Atkinson {1970 : p.253}

<sup>2</sup>Srinivasan, Bardan et al. {1974 : p.80}

- (9) The Relative Mean deviation,
- (10) Theil's Entropy measure,
- (11) Dalton's measure of inequality,
- (12) Sen's (generalised) Index.

In the choice of suitable measures of inequality for purposes of this study, it is relatively easy to exclude measures (6) - (12). For, measures (6) and (7) are no more than special cases of the Atkinson Index.<sup>1</sup> For example, by setting  $\epsilon = -1$  in the Atkinson Index, we get

$$I = 1 - \left[ \frac{1}{n} \sum \left( \frac{y_i}{\mu} \right)^{-1} \right]^{-1} \text{ for the}$$

simpler case of an array of values  $y_1$  to  $y_n$

$$\begin{aligned} \text{i.e. } I &= 1 - \frac{1}{\frac{\mu}{n} \sum \frac{1}{y_i}} \\ &= \frac{1}{\mu} \left[ \mu - \frac{1}{\frac{1}{n} \sum \frac{1}{y_i}} \right] \\ &= \frac{1}{A.M} [A.M - H.M.] \end{aligned}$$

i.e. Measure (7)

Although the Pareto constant and Pareto's 'law' of income distribution were the earliest to be proposed, their limitations were well known. For, Pareto's law yields

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<sup>1</sup>See Champernowne {1974 : pp. 787 - 816}

satisfactory results only in respect of the upper income groups in an income distribution and not for the distribution as a whole. An interesting discussion of Pareto's law is found in Pen {1971 ; pp.234-6.} The main difficulty with the relative mean deviation is that it is not at all sensitive to transfers from the poorer person to a richer person as long as both lie on the same side of the mean. For, as illustrated by Sen {1973 : p.26}, if transferred from the poorest man to some one more rich but having less than the mean income would produce equal and opposite results so as to leave the measure unchanged.

Dalton's index of inequality may be expressed by

$$D = \frac{\int_0^{\bar{y}} U(y)f(y)dy}{U(\mu)} \quad (2.6)$$

The first problem encountered here is that of specifying the form of the utility function  $U(y)$ . The main difficulty, as pointed out by Atkinson {1970 : p. 249} is that  $D$  is not invariant with respect to linear transformation of the function  $U(y)$ . He illustrates this by taking the case of the logarithmic utility function. In this case, although two people might agree that the social welfare function should be logarithmic, their measures of inequality would coincide only if they also agree on the constant term in the log function.

Sen's (generalised) Index (12) may be regarded as a generalization of the Atkinson Index. Sen's Index is based on social welfare function:

$$W = W(y_1, \dots, y_n), \quad (2.7)$$

If the generalised equally distributed equivalent income  $y_f$  is defined as that level of per capita income which if shared by all will produce the same  $W$  as the value of  $W$  generated by the actual distribution of income, then

$$\text{Sen's Index } N = 1 - \left( \frac{y_f}{\mu} \right) \quad (2.8)$$

Sen has pointed out that this measure reduces to the same as the Atkinson Index if  $W$  is specified as

$$W = \sum_{i=1}^n U(y_i) \quad (2.9)$$

The main difficulty associated with this measure is one of specifying the form of the welfare function. As would be seen shortly, there is a much simpler way of specifying our notion of welfare in the case of the Atkinson Index.

Theil's entropy measure is given by

$$T = \sum_{i=1}^n x_i \log nx_i \quad (2.10)$$

where  $x_i$  = share of income of person  $i$ .  
(see Sen {1973 ; p.35})

Although this measure of inequality has a useful analogy with the notion of the inexorable trend towards entropy in thermodynamics, yet as pointed out by Sen {1973 ; p.36} the

form of the function in Theil's formula does not quite correspond to our intuitive notions of utility.

The range for selection thus gets limited to measures (1) - (5). Although measure (1) i.e C is one of the simplest measures which captures total variation, it has a number of drawbacks. For example, the squaring procedure is somewhat arbitrary, and so is the selection of the mean for measuring of deviations. The main drawback is that all income transfers are given equal weightage irrespective of whether they are taking place at the upper end or lower end.

Sen's measure of Poverty is of great relevance to inequality measurement in developing countries, where elimination of poverty is a major objective. However, on account of certain difficulties in selecting a poverty line, we shall avoid the use of this index. Instead, we shall briefly note some of the results already obtained in respect of Sri Lanka by application of this measure. We are thus left with measures (2), (3) and (4) for which there are several arguments in favour. Although measure (2) is also characterised by an arbitrary squaring procedure and differences are taken from the logarithm of the mean, yet it is frequently used because of one important feature. That is, if we wish to attach greater importance to income transfers at the lower end, then we should select a transformation of incomes that staggers



the income levels. One such transformation is the logarithmic transformation (see Sen {1973 ; pp.28-29}). Although this measure is claimed to be a positive measure (there is no explicit welfare consideration), its frequent selection supports our earlier point about the distinction between positive and normative measures being not so clear cut. For, its frequent selection is based on a welfare consideration implicit in it, namely the weightage given to income transfers at the lower end.

The Gini Coefficient is perhaps the best known and most popular amongst measures of income inequality. Its popularity is due to several reasons. Firstly, the arbitrary squaring procedure is avoided. Secondly, differences are taken between every pair of incomes. More importantly, it has a well known interpretation in terms of the Lorenz curve. It could be shown to be equal to the area between the line of equal distribution and the Lorenz curve divided by the area of one of the triangles of the square separated by the diagonal. But as in the case of the previous measure, although the Gini Coefficient is supposed to be a positive measure of inequality, it has a welfare consideration implicit in it.

As shown by Sen {1973 ; pp.31-33}, the Gini Coefficient G could be written as

$$G = 1 + \frac{1}{n} - \frac{2}{\mu n^2} (y_1 + 2y_2 + \text{-----} + n y_n)$$

$$\text{for } y_1 \geq y_2 \text{ -----} \geq y_n$$

Thus, there is a welfare consideration implicit in G because the third term on the right has a weighted sum of incomes, the weights being determined by the rank order of the people with these incomes.

The Gini Coefficient and the Lorenz curves, apart from being useful in empirical work, have also stimulated a considerable amount of theoretical interest. Atkinson has proved an important theorem regarding the ranking of two distributions according to levels of welfare in terms of the respective Lorenz curves. Let us suppose that A and B are two distributions of the same total income and that  $L_A$  and  $L_B$  are the respective Lorenz curves. Let us also suppose that  $L_A$  is wholly inside  $L_B$ . Then, Atkinson {1973 ; pp. 245-247} has proved that the level of welfare associated with  $L_A$  is greater than that associated with  $L_B$  subject only to the assumption that utility as a function of income is concave i.e.  $U'(y) > 0$  ,  $U''(y) < 0$  .

## LORENZ CURVES

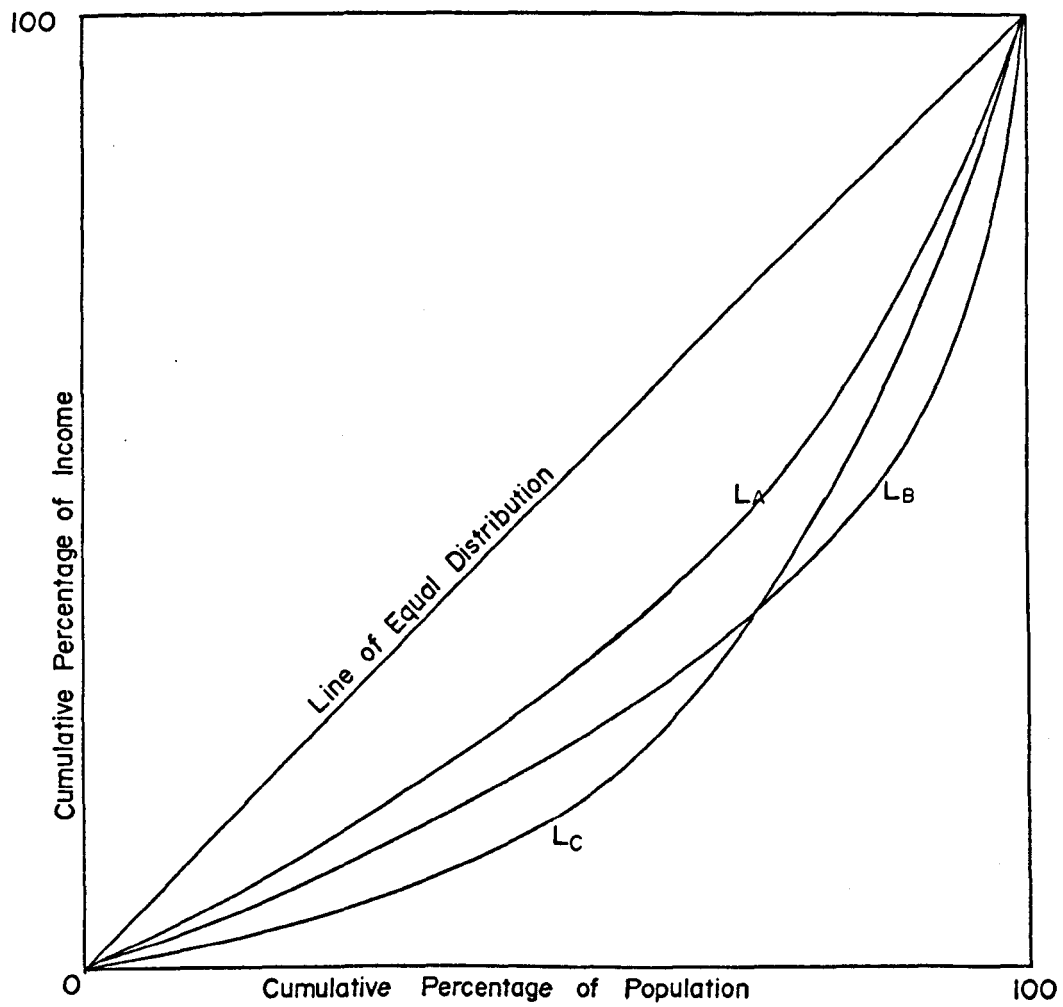


Figure 2.1

In proving this theorem it was shown that there was no necessity to assume any particular form of the welfare function. But if two Lorenz curves intersect as in the case of  $L_B$  and  $L_C$ , no conclusion could be reached as to which distribution would have a higher level of welfare, without an explicit welfare assumption. That is, the areas between the  $L_B$  and the diagonal and  $L_C$  and the diagonal may be equal i.e the Gini Coefficients may be equal, but yet the two distributions could represent two levels of welfare. This is in fact one of the shortcomings of the Gini Coefficient, and <sup>it</sup> points out to the inevitability of normative considerations.

The Atkinson Index is based on the concept of an "equal distributed equivalent income" (Yede) which is the level of per capita income which if shared equally will give the same level of social welfare as given distribution. Denoting income by  $y$ , frequency by  $f(y)$  and utility from income by  $U(y)$ , Yede is defined implicitly (see Atkinson {1970 ; p. 250})

$$U(Yede) \int_0^{\bar{y}} f(y) dy = \int_0^{\bar{y}} U(y) f(y) dy \quad (2.11)$$

The Atkinson index is then defined as

$$I = 1 - \frac{Yede}{\mu}, \text{ where } \mu = \text{mean income.} \quad (2.12)$$

As in the case of the Gini Coefficient,  $I = 0$  implies complete equality and  $I = 1$  implies complete inequality. Assuming  $I$  to be invariant with respect to proportional shifts in income, it could be proved (see Codippily {1974 ; p.12-14} ) that

$$I = 1 - \left[ \sum \left( \frac{y_i}{\mu} \right)^{1-\epsilon} f(y_i) \right] \frac{1}{1-\epsilon} \quad (2.13)$$

The main advantage of the Atkinson Index is that our welfare considerations could be narrowed down to assigning a value to  $\epsilon$ . A higher value for  $\epsilon$  implies a greater weightage to transfers of income at the lower end and a lower value of  $\epsilon$  implies lower weightage. The parameter  $\epsilon$  is thus a measure of the degree of inequality aversion. Once a decision is made about the value that could be assigned, then ambiguities do not arise as in the case of the Gini Coefficient.

We have thus chosen three measures of income inequality namely the standard deviation of logarithm, the Gini Coefficient and the Atkinson Index. It is interesting to note that all three measures satisfy the criteria for good indices set out by Champernowne {1974 ; pp. 787-816} , namely:

- (a) Familiarity and convenience for computation from statistics readily available;
- (b) Impartiality between persons;
- (c) Invariance with respect to the number of persons, i.e. being unaffected so long as proportions of distribution between

groups are unaffected;

- (d) Invariance with respect to uniform increase;
- (e) The Pigou-Dalton criterion i.e if a distribution is modified by altering two incomes so as to leave the total the same, then the index of inequality must be increased, unchanged or decreased according as the absolute difference is increased unchanged or decreased.
- (f) Range from 0 to 1; and
- (g) Suitability for type of inequality discussed.

All three measures discussed above have been used in evaluating the extent of inequality in each sector as well as in the island as a whole.<sup>1</sup> The results obtained by using the first two measures were as follows:

Table 2.6

Income inequality amongst Spending Units  
classified by Sector - 1973

Sector	Std. Dev. of Logarithm	Gini Coefficient
Urban	0.743	0.375
Rural	0.643	0.332
Estate	0.572	0.299
All Island	0.648	0.350

Both measures indicate that income inequality is lowest in the estate sector, and highest in the urban sector. An

<sup>1</sup> The calculations were done with the assistance of a computer programme made available to me by Mr. Anthony Flegg.

immediate point of interest is to inquire as to how income inequality could have changed over the ten year period 1963-73. The extent to which income inequality has changed over this period could be seen from the table below:

Table 2.7

Changes in Income Inequality amongst Spending  
Units classified by sector

Sector	Gini Coeff. 1963	Gini Coeff. 1973
Urban	0.481	0.373
Rural	0.424	0.332
Estate	0.301	0.299
All Island	0.450	0.350

Results in the above table are in general agreement with those of Table 2.5. That is, substantial reductions in inequality are witnessed in the urban and rural sector. But there has been no appreciable change in the case of the estate sector. These changes could be seen more clearly in terms of Lorenz curves appearing in Figures 2.2 to 2.5.

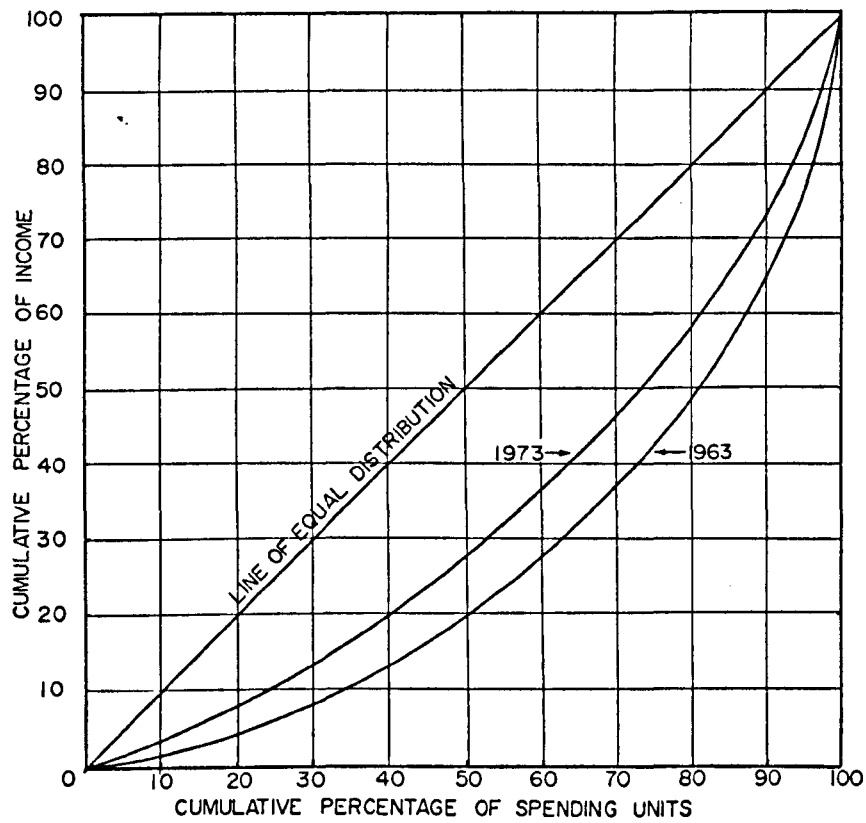
LORENZ CURVES FOR SPENDING UNITS - ALL ISLAND, 1963 & 1973

Figure 2.2

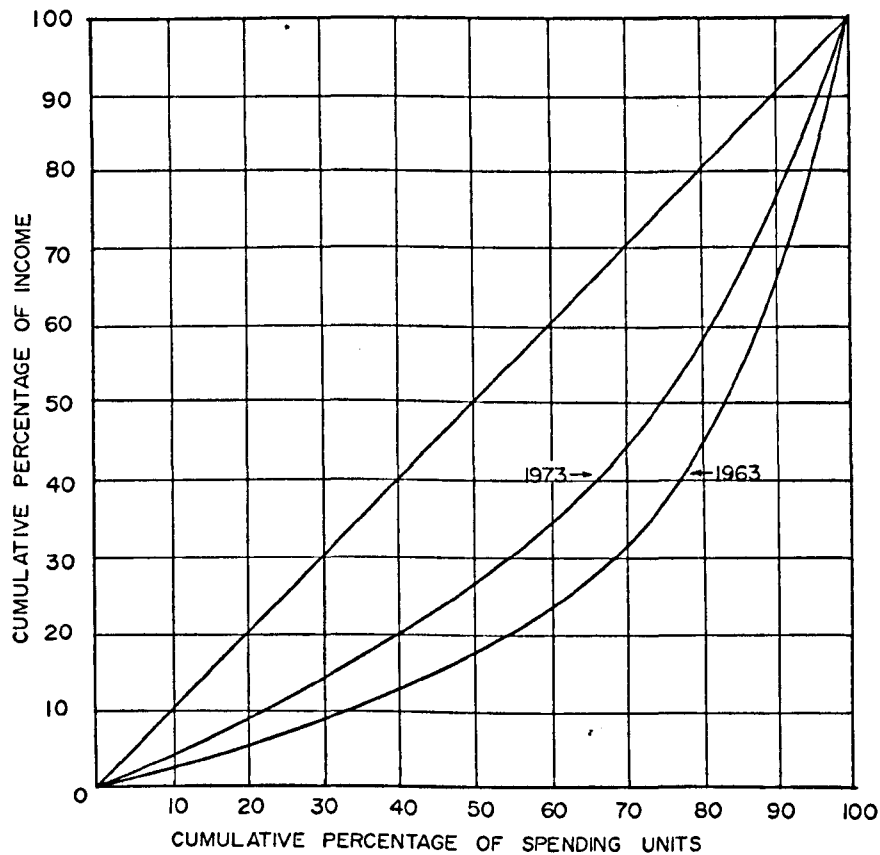
LORENZ CURVES FOR SPENDING UNITS - URBAN SECTOR, 1963 & 1973

Figure 2.3



# LORENZ CURVES FOR SPENDING UNITS- RURAL SECTOR, 1963 & 1973

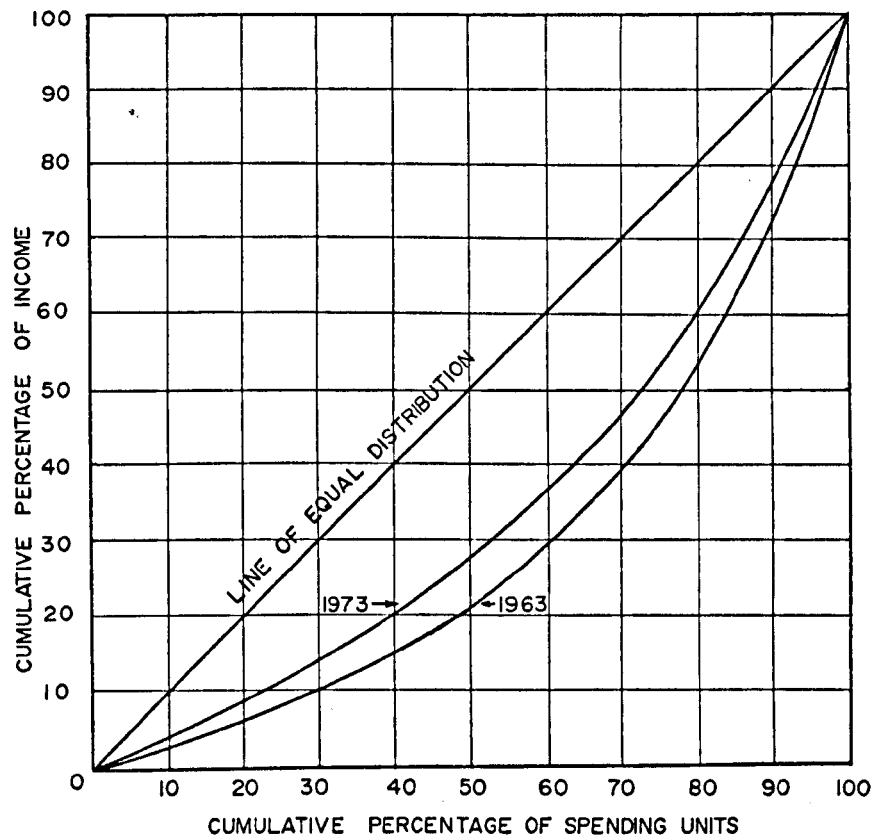


Figure 2.4

# LORENZ CURVES FOR SPENDING UNITS- ESTATE SECTOR, 1963 & 1973

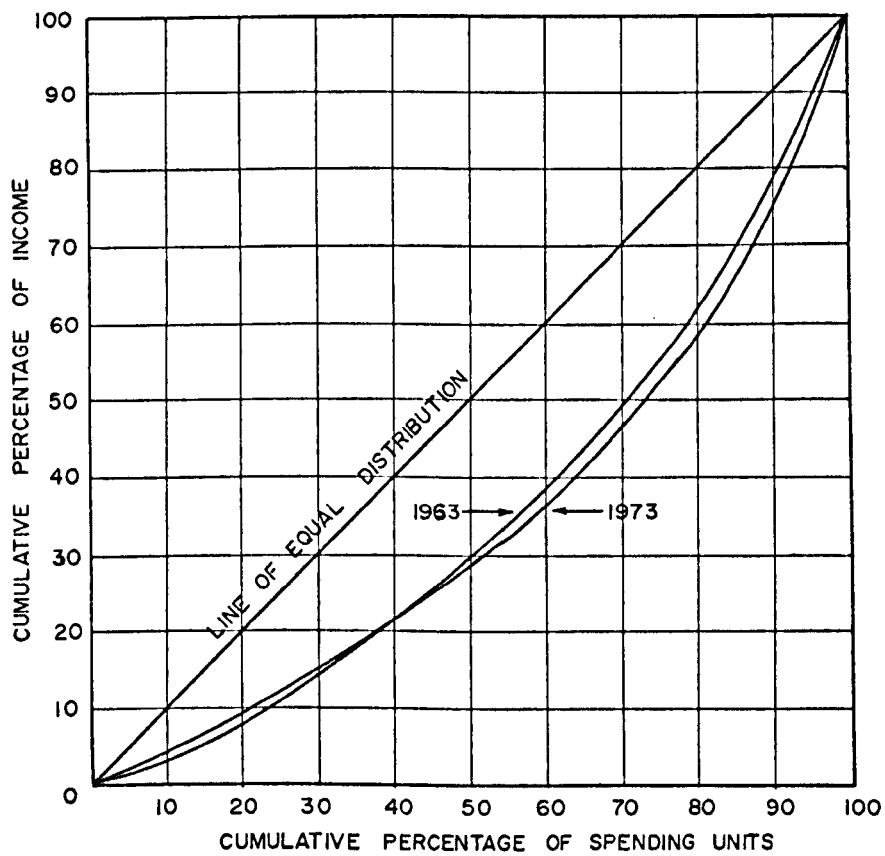


Figure 2.5

Figures 2.2 to 2.4 express in visual terms the manner in which income inequality has declined in the urban sector, rural sector and the island as a whole. But in the case of the estate sector, Fig. 2.5 does not indicate any clear cut change in income inequality; the movement of the Gini Coefficient from 0.301 in 1963 to 0.299 in 1973 is insignificant. This example indicates the limitations of the Gini Coefficient and the Lorenz curve and points out to the inevitability of a welfare assumption. Particularly in view of the slight improvement of the share of the lowest quintile it will be of interest to calculate the change in inequality in terms of an index which is based on an <sup>explicit</sup> welfare consideration, such as for example the Atkinson Index. But before we proceed to do so, we should briefly note the results obtained in respect of the changes in the Gini Coefficient of Income receivers. As reported by Karunatilake {1974 ; p.105} the changes in income inequality amongst income receivers could be seen from the table below:

Table 2.8

Income Inequality amongst Income Receivers  
classified by Sector 1963 & 1973

SECTOR	Gini Coeff. 1963	Gini Coeff. 1973
Urban	0.49	0.40
Rural	0.44	0.37
Estate	0.27	0.37
All Island	0.49	0.41

Whilst the results in the above table in respect of the urban sector, rural sector and the island as a whole are in agreement with those of Table 2.7, income inequality in the estate<sup>sector</sup> has shown a marked increase. In contrast, when spending units are considered, no such deterioration is seen. A possible explanation for this disparity could be the changes in the number of income receivers per spending unit particularly in the lower income groups. There is no clear cut way of testing this hypothesis. Although information regarding the number of income receivers per spending unit classified by income groups could be derived from CFS 1973, the CFS 1963 does not lend itself to such an analysis.

Let us now proceed to the results obtained in regard to income inequality by the application of Atkinson's Index. The results obtained are presented in the table below:

Table 2.9  
The Atkinson Index in respect of Urban  
Rural & Estate Sectors-1973

Sector	Atkinson Index for			
	$\epsilon = 1.0$	$\epsilon = 1.5$	$\epsilon = 2.0$	$\epsilon = 2.5$
Urban	0.216	0.309	0.403	0.513
Rural	0.173	0.249	0.324	0.405
Estate	0.144	0.208	0.272	0.342
All Island	0.174	0.251	0.330	0.416

Following Atkinson's procedure {1970 ; p.258} if we rank

income distributions independent of mean income levels, then the above table shows that the rankings of inequality as between the urban, rural and estate sectors are the same as those obtained by the Gini Coefficient and the standard deviation of the logarithms. But the advantage of using the Atkinson Index is that it helps us to clarify the case of the intersecting Lorenz curves shown in Fig. 2.5. If our concern towards the poor is high and we assign a value of 2 to the 'measure of inequality aversion'  $\epsilon$ , then the Atkinson Index for the Estate Sector (in 1963) works out to 0.312. The 1973 index is 0.272 (from the above table) and thus a decline in inequality is shown. It should be noted that this result is based upon our subjective notion of welfare, which attaches importance to income transfers at the lower income levels. By assigning a high value of  $\epsilon = 2$ , the effect produced is that of magnifying the increase in the share of income obtained by the bottom 30 per cent or so.

An interesting application of Sen's Poverty Index on Sri Lanka data has been carried out by Radhakrishnan {1976}. By assuming a poverty line of Rs.100 per month, per capita at 1974 prices, he showed that the Poverty Index declined from 0.24 in 1963 to 0.11 in 1973 indicating a substantial improvement in the standard of living of the poorer sections of the population.

Whilst attention has been focussed in the above discussion on inequality within each sector, the latter part of the previous section dealt with income inequality between sectors. An interesting approach to discuss both these aspects of income inequality in terms of a disaggregated Gini Coefficient has been made by Pyatt {1976}. Although a similar approach had been made by Bhattacharya and Mahalanobis ( as reported in Pyatt {1976}), yet the main appeal in Pyatt's approach is that of providing simpler proofs and interpretations in terms of statistical game theory. Having expressed the Gini Coefficient as

$$G = \frac{(1/n) \sum_{i=1}^n \sum_{j=1}^n \max(0, y_i - y_j)}{(1/n) \sum_{i=1}^n y_i} \dots \quad (2.14)$$

it is shown that the Gini Coefficient is the average gain to be expected if each individual has the choice of being himself or some other member of the population drawn at random, expressed as a proportion of the average level of income. This game theory interpretation is extended in an interesting manner to the case where a population is divided into a number of groups, and in this case,

$$\text{average expected gain} = \sum_{i=1}^k \sum_{j=1}^k E(\text{gain}/i \rightarrow j) \text{Pr}(i \rightarrow j)$$

where "  $i \rightarrow j$  " refers to an individual being in the population group  $i$  and drawing a member of group  $j$  to compare himself with in the hypothetical game.  $E(\text{gain} / i \rightarrow j)$  is the average gain if they draw a member of group  $j$  for

comparison. The results are combined in terms of the matrix equation

$$G = (\underline{m}' \underline{p})^{-1} \underline{p}' E \underline{p}$$

where  $\underline{p}$  and  $\underline{m}$  are both  $k$  - element column vectors,

and  $i^{\text{th}}$  element of  $\underline{p}$  = population proportion  $p_i$

$i^{\text{th}}$  element of  $\underline{m}$  = average income of individuals  
in population group  $i$ .

By means of the main theoretical result obtained (see Pyatt {1976 ; p.249, equation (21)}), Pyatt shows that  $G$  could be represented in two parts: the first arising from variation within groups and the second depending entirely on differences between group means. In an empirical exercise concerning income inequality amongst income receivers in Sri Lanka in 1973, Pyatt {1976 ; p.248} shows that the latter component (i.e relating to differences between groups) contributes only 0.12 to the aggregate Gini Coefficient of 0.41.

## 2.5 Summary

To sum up, the main conclusion of the discussions in this chapter is that significant progress has been made in moving towards a more equitable distribution of incomes in Sri Lanka over the period 1963-73. The share of income of the highest decile declined sharply from 40.60 per cent in 1963 to 28.03 per cent in 1973, whilst the share of the lowest decile improved significantly from 1.50 per cent in 1963 to 2.79 per

cent in 1973. In the case of the other lower deciles too, significant improvements in the shares of income have taken place. The analysis of the patterns of income distribution in urban, rural and estate sectors shows that the overall reduction in income inequality had come about mainly from reduction of income inequalities in the urban and rural sectors. There could have been no contribution from the estate sector towards the reduction of income inequality. For, positive measures of inequality do not indicate any appreciable change in the level of inequality in the estate sector over the period 1963-73.

The other interesting set of results examined were concerning the disparities in levels of income and consumption as between urban, rural and estate sectors. Noteworthy results obtained were those of an increase in the share of non-urban incomes although the proportion of non-urban population had declined. In real terms, non-urban incomes per spending unit have witnessed steady rates of growth in contrast to a decline in the case of urban spending units. Results indicate that conditions have definitely moved in favour of the non-urban population. As at 1973, except in the case of a few selected items, there were no marked disparities in <sup>average</sup> consumption as between urban, rural and estate spending units. The items in respect of which disparities existed were the relatively higher level of consumption of protein foods in the urban

sector, inadequate housing facilities in the estate sector, low expenditure on education by estate spending units, and the relatively higher expenditure on durable consumer goods by the urban spending units.

An attempt was also made in this chapter to look at how the absolute levels of income accruing to the various income groups have changed over time. It was noted that real income had increased by 51.1 per cent over the period 1963-73. When this change was disaggregated, it was evident that results of growth had reached the lower income groups - a little less than a third of the total <sup>absolute</sup> increase in real income had reached the two lowest quintiles in the urban, rural and estate sectors. That is, there has been a growth and redistribution of incomes.

Inter-country comparisons made in this chapter have shown that the pattern of income distribution in Sri Lanka in 1973 had been very favourable in relation to those prevailing in other countries. One aspect that was not covered in this chapter was the situation after 1973. The impact of the global recession, the oil crisis and three successive droughts could have not only worsened the real incomes of certain income groups but could have also had the effect of increasing income inequality. The fixed income earners, the self employed, and the urban poor are probably the worst affected by the rise in domestic prices arising from these unforeseen developments. However, no survey



results are available in respect of the post 1973 period. A survey of household expenditure patterns is just being initiated and the results are likely to be available by mid 1979. The results of this survey could throw some light on the changes after 1973.

The pattern of income distribution that prevailed in 1973 has been brought about through a number of conscious policy measures by successive governments. A review of these policy measures will be the main theme of the next chapter.

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CHAPTER 3A REVIEW OF REDISTRIBUTIVE POLICIES

"..... economic welfare is best promoted by a minimum standard raised to such a level that the direct good resulting from the transference of the marginal pound transferred to the poor just balances the indirect evil brought about by the consequent reduction of the dividend" - A.C. Pigou (The Economics of Welfare, page 761).

### 3.1 Introduction

The aim of this chapter is to complement the results of the preceding chapter by identifying and discussing some of the factors that could have contributed to the reduction of income inequality in recent years. Discussions in this chapter are to a large extent based on existing surveys on the subject. Attempts will be made wherever possible to isolate the redistributive character of these factors.

We noted in Chapter 2 how the country's plantation sector began to evolve during the British period. The development of the plantation sector together with its infrastructure and supporting services had led to the formation of a dual economy of the classic type - a large rural subsistence sector side by side with a

small but powerful modern sector based on export oriented plantation activities. This dual economy, as described by the Marga study {1974} "had produced a dual society which was separated by language, attire, ways of living and outlook". The modern sector had given rise to a relatively affluent, English educated urban elite who occupied the more privileged positions in society. In contrast, the vast majority of the "common" folk who were mostly rural, were dependent upon the traditional sector for employment and were at a subsistence level. They had little or no access to the privileges enjoyed by the urban elite<sup>1</sup>.

One of the first measures which sought to rectify this imbalance was the introduction of free education in 1945 and adoption of the mother tongue as the medium of instruction in schools. The main objective was to provide greater equality of opportunity to the various segments of society to pursue their aspirations. The other important welfare measures introduced in the 1940s were the food subsidy, free health services, minimum wage policies and subsidised public transport. These measures were further strengthened after Independence in 1948 by the government which had by then assumed the role of a benefactor. Short

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<sup>1</sup> See Marga Study {1974; pp.3 - 7} for a fuller discussion

term political objectives were closely interwoven with welfare policy during the post Independence period. Welfarism became part and parcel of the political philosophy of the major political parties, and was seen as a means of winning or maintaining power over the electorate. Expenditure on welfare services tended to increase with each successive government, so that by the mid 1960s, welfare expenditure amounted to as much as 57.6 per cent of government expenditure. During the same period, e.g 1962-65, the corresponding figures in the neighbouring countries were 24.6 percent for Thailand, 24.4 percent for Malaysia and 18.1 percent for India (see Lotz {1970}).

Another major factor that is considered to have contributed towards reduction of income inequality in recent years is the programme of agricultural development carried through during the post-Independence period. Motivated by import substitution, large scale agricultural development schemes were initiated during the post-Independence period. Increased farmer incomes which was a concomitant of these developments is considered to have contributed towards reduction in income inequality, particularly in the rural sector.

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### 3.2 The Food Subsidy

The origins of the present food subsidy could be traced back to the time of the Second World War when a rice ration had been introduced at a subsidised price. This had been done in order to cushion the consumer from price fluctuations arising from the shortages of food supplies due to the war. The food subsidy thus initiated continued into the post-war period and even into times during which (early 1950s) there were no major food scarcities. In view of the increasing strains that were being placed upon the government budget, attempts were made from time to time to reduce the subsidy. But these attempts were met with strong political opposition as for instance the agitation and unrest of the working class 'hartal' in 1952. Since then the major political parties have been committed to the continuation of the rice subsidy, subject to certain modifications from time to time.

The rice subsidy is the major element in the total food subsidy. For example in 1975, the rice subsidy constituted 64 percent of the gross food subsidy while flour and sugar subsidies accounted for 18 and 17 percent respectively<sup>1</sup> and the subsidy on infant milk represented less than 1

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<sup>1</sup> Source: Central Bank of Ceylon, Review of the Economy 1975, p.158.

percent. In the past, certain items, notably flour and sugar were sold at a profit so as to reduce the total subsidy and to yield a net subsidy which was slightly less than the gross subsidy. But in the years 1974 and 1975 these two items were subsidised rather than being sold at a profit following the sugar crisis and the rise in price of flour.

The rice subsidy, in its history of 35 years or so, underwent several modifications from time to time. In the mid 1960s the subsidy took the form of providing each person with 2 measures of rice <sup>per week</sup> (1 measure = 2 pounds) at a subsidised price. In 1966 a significant change took place, namely, that one measure was provided free of cost and the other at market prices. With the change of government in 1970, the second measure was restored at a subsidised price on ration and the free measure was continued. (see Mahalingasivam{1978}). But in 1972 the supply of free rice to tax payers was stopped. On account of the worsening balance of payments situation that arose in 1973 due to the OPEC price increase for crude oil, the free measure had to be halved; each person in a non-tax paying household received per week, half a measure of rice (one pound) free and one measure of rice at a price of Rs.2.00 which was slightly below the market price. With the change of

government in July, 1977, a major shift away from consumer subsidies was seen. This was announced categorically in the Budget Speech of November, 1977 as:

"the country has for much too long relied on a proliferation of subsidies of one kind or another which by pre-empting resources for consumption have severely limited progress towards development and employment".

Specifically, the proposal was to remove the rice subsidy from households receiving an income of over Rs.300 per month in money terms. This scheme is operative at present.

In the case of sugar, it had long been an item sold at a profit so as to reduce the food subsidy at least marginally. But, following the sugar crisis in 1973/74, a subsidy was introduced so as to provide a basic minimum at a subsidised price. One pound of sugar was provided on ration per person per week at 72 cents and the balance at a market price of Rs.5 per pound. As proposed in the 1977 Budget, this subsidy too has been removed. Sugar is now sold in the open market at Rs.3 per pound. Flour too is subsidised and is supplied through the consumer co-operatives and other outlets at Rs.1.05 per pound. The present subsidy on flour is approximately 70 cents per pound.

Table 3.1

Subsidies as percentages of total  
income of Spending Units

Income Group (Monthly Income)	<u>Subsidies as a percentage of Total Income</u>			
	Rice	Flour	Sugar	Total
Less than Rs.25	9.63	4.68	2.73	17.04
Rs.26 - Rs.50	10.60	1.92	3.48	16.00
Rs.51 - Rs.100	8.35	2.07	2.13	12.55
Rs.100 - Rs.200	8.84	2.70	2.12	13.66
Rs.201 - Rs.400	6.85	2.07	1.70	10.62
Rs.401 - Rs.800	4.05	1.00	1.13	6.21
Rs.801 - Rs.1000	1.64	0.65	0.56	22.85
Rs.1001 - Rs.1500	0.81	0.31	0.36	1.48
Over Rs.1500	0.31	0.17	0.20	0.68
Total	5.49	1.60	1.40	8.44

Source: Derived from CFS 1973 Table 84 p. 99.

As seen from the above table, the food subsidy has accounted 16 - 17 percent of the incomes of the lowest income groups and would have contributed positively towards reduction of income inequality.

### 3.3 Health Services

The development of an extensive system of health services has been an important component of social welfare policy during the past 30 years or so. In the 1940s, it took the form of an



Anti-malaria campaign, establishment of rural hospitals, development of the Colombo Medical College and other institutions, improvement of existing hospital facilities and the establishment of a wide range of other services. For example, as a result of one such service, today nearly all births are professionally supervised.<sup>1</sup> As at 1972, there were 2,253 western type government hospitals, dispensaries and institutions, 218 indigenous "Ayurveda" hospitals and dispensaries, 787 western type private nursing homes, hospitals and practitioners and nearly 20,000 private Ayurveda and other practitioners, making a total of a little over 23,000 "institutions" in all.<sup>2</sup> The health services in Sri Lanka today ranks amongst the best in Asia. Table 3.2 attempts to present the comparative position regarding basic facilities.

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<sup>1</sup>See Jones and Selvaratnam{1972;Table 12.} In 1966, 99 per cent of births were professionally supervised as compared with Singapore (99), Hong Kong (95), Jamaica (70), Venezuela (61), U.A.R (35), Ghana (33), Peru (28), Malaysia (28), Philippines (23), and Thailand (16).

<sup>2</sup>Source: World Health Organisation. Better Health for Sri Lanka, Report on a Health Manpower Study by Dr. L.A. Simeonov (SEA/PHA/149) p.8, 1975.

Table 3.2An Inter-country comparison of Medical Facilities

Country	Year	Population per Hospital Bed	Year	Population per Physician
1. Bangladesh	1972	6,946	1973	9,345
2. India	1968	1,571	1973	4,162
3. Japan	1972	78	1973	868
4. Republic of Korea	1973	1,651	1974	2,571
5. Malaysia (Sabah)	1973	348	1973	8,941
6. Malaysia (West)	1972	276	1971	630
6. Pakistan	1973	1,853	1973	4,049
7. Philippines	1969	822	1970	2,639
8. Singapore	1973	269	1973	1,399
9. Sri Lanka	1973	333	1972	4,007
10. Thailand	1973	774	1973	n.a

Source: U.N. Statistical Year Book 1975. Table 208 p.841.

Efforts in developing health services have produced significant results. For, the death rate which stood at 20.2 per thousand in 1946 declined to 7.9 per thousand in 1972; infant mortality declined from 141 per thousand live births in 1946 to 50 per thousand in 1968 the expectation of life at birth increased from 44 for males and 42 for females in 1946 to 65 and 67 respectively in 1968.<sup>1</sup> The expectation of life at birth, particularly in regard to males, comes close to the corresponding figures in developed countries. For example, around 1973/74, the corresponding figure for males was 67.4

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<sup>1</sup> See Chapter 2, Section 2.1

years in U.S.A, 67.8 years in U.K, 72.1 years in Sweden and 70.5 years in Japan.<sup>1</sup>

Unlike in the case of the food subsidy, there is no quantitative information which will enable us to assess the extent to which each income group has benefited from the health services. But it is well known that it is because free medical services were available that the lower income groups had access to modern medical facilities. The benefits are two fold. Firstly, the value of these services must necessarily be regarded as a part of the real income of those receiving medical facilities and secondly better health would have contributed in some measure towards raising the level of productivity and income of the low income workers. In regard to the former, the total government expenditure during 1973 had been Rs.298.3 million.<sup>2</sup> which averages to Rs.110.3 per spending unit per annum. Even if we assume that both rich and poor spending units have received this benefit equally on the average, it is clear that such an addition to income would have reduced income disparity, in some small measure. However, it is more likely that the lower income groups

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<sup>1</sup> Source: U.N. Statistical Year Book 1975, Table 19.

<sup>2</sup> See U.N. Statistical Year Book 1975, Table 201, p.789.

would have benefited more, in which case, the argument is further strengthened. This possibility could be supported by an assessment made in the WHO study {1975}. As shown in this study, in 1969/70, households receiving incomes less than Rs.200 per month were able to spend on an average of Rs.2.05 per month on private health services. In contrast, households receiving over Rs.1000 per month had spent an average of Rs.20.02 per month on private health services. In these circumstances, low income households would have had to rely more on state medical services.

#### 3.4 The Educational System

The third major component of social welfare policy in Sri Lanka during the past 30 year period was the Educational System adopted. In 1945, three years before Independence, a free education policy was adopted to provide free education from the Kindergarten to the University level, and by 1951 this policy had been fully implemented. In the Preliminary Survey of Education prepared by the World Bank in 1966, Sri Lanka's educational system was described as follows:

"To judge from its educational pyramid alone it might be said that Ceylon, after Japan, had the best developed education system in Asia. A very high proportion of children attend school, the wastage rates are relatively low, the numbers of girls are little lower than of boys; schools at secondary level are well attended and very large numbers take

the school certificate examination at the end of the tenth year. It is true that enrolments in pre-university classes and at the universities are low and the output of graduates small, but even here enrolments have recently increased greatly. Moreover, Ceylon has provided this education free at all levels, so that it is not surprising that the proportion of the GNP spent on education is almost 5 per cent and the highest in Asia".

As at 1973, the total student enrolments stood at 1.539 million at primary level, 1.150 million at secondary level and a little over 19,000 at tertiary level. Thus, total enrolments in 1973 represented 20.5 per cent of the population. The total teaching staff in 1973 exceeded 100,000.

In the 1970s, the percentage of GNP spent on education had been somewhat lower, as for example 4.3 per cent in 1973. Nevertheless Sri Lanka still ranks amongst the highest in Asia, in terms of percentage of GNP spent on education as could be seen from the Table 3.4.

Government expenditure on education has shown a steady increase over time. From a level of Rs.602.0 million in 1973 it increased to Rs.622.8 million in 1974 and to Rs.718.3 million in 1975. However as a percentage of total government expenditure, it has declined from 12.7 per cent in 1973 to 11.3 per cent in 1974 and to 10.9 per cent in 1975.

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Table 3.3

Expenditure on Education as a percentage of the  
Gross National Product

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Country	Year	Expenditure of Education as of GNP
1. Burma	1971	3.4
2. India	1972	2.5
3. Japan	1972	4.3
4. Korea	1973	3.0
5. Malaysia (West)	1971	5.1
6. Pakistan	1973	2.0
7. Philippines	1972	2.0
8. Singapore	1973	2.7
9. Sri Lanka	1973	4.1
10. Thailand	1973	3.0

Source: U.N. Statistical Year Book 1975,  
Table 210.

The results of these efforts have today taken the form of a high rate of literacy and a relatively wide spread of basic education. As noted in the previous chapter the literacy rates among males and females in 1971 were 85.2 and 70.7 respectively. Further, from the population of persons over 15 years, 30.3 per cent had received an education up to GCE (O.L) and 1.5 per cent had passed the GCE (A.L) or higher examinations.

It could also be inferred from the CFS 1973 that improvement in educational standards over the period 1963 - 73 would have contributed towards reduction in income inequality. According to Table 13 of the CFS 1973, during the period 1963-73, the Index of Education attained<sup>1</sup> had changed from 5.15 to 5.40 in the case of the urban sector, 3.94 to 4.60 in the case of the rural sector, and 2.09 to 2.31 in the case of the estate sector. Again, Table 60 of CFS 1973 shows how mean and median incomes increase with increasing educational levels. It would therefore appear that the relatively high increases in the Index of Education attained in the case of the rural sector could have contributed towards raising rural incomes. This in turn could have contributed towards reducing the urban-rural income disparities.

### 3.5 Other Welfare Measures

Several other welfare measures were also implemented during the past 30 year period. Without going into detail, we may briefly note some of the salient features of the more important ones, namely minimum wage policy, subsidies on public transport and price control.

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The "Index of Education Attained" had been computed by weighting each educational standard achieved (e.g. No schooling, literate, Primary, Secondary, Passed GCE/SSC etc.) by the minimum number of years necessary to attain it, and obtaining the weighted average. A detailed explanation is provided in CFS 1963 Table 10.

The present minimum wage policies originated from the Wages Board Ordinance of 1941. This Ordinance, together with its nine subsequent amendments provide the legal framework for fixing minimum wages for the various trades. In each Wages Board set up under this Ordinance there are an equal number of representatives from the employers and employees and a maximum of three members nominated by the Government. Each Board fixes the minimum wage applicable to the particular trade they are concerned with. At present there are 32 trades covered by minimum wage legislation.<sup>1</sup> It is also estimated that nearly one and a half million workers are covered by minimum wage legislation, representing 67 per cent of those contributing to the Employees Provident Fund or nearly 40 per cent of the total employed.

Likewise, in the case of those working in shops and offices, the Shop and Office Employees Act No. 19 of 1954 sought to regulate their terms of minimum remuneration and the grant of Public Holidays. This Act made provision for the determination of remuneration by the Commissioner of Labour where employers and employees consent to such determination and for the setting of Remuneration Tribunals for the determination of remuneration in specified shops and offices (see Weerakoon {1976} ).

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<sup>1</sup> A fuller discussion is provided by G. Weerakoon in the Sri Lanka Labour Gazette of January, 1976.



One would expect the results of minimum wage policy together with the trade union movement to be reflected in a reduction of income disparities as between professions over the period 1963-73. But this unfortunately is a comparison we cannot make on account of major differences in the classifications of professions as between the CFS 1963 and the CFS 1973.<sup>1</sup>

The lower income groups are also considered to have benefited from subsidised public transport. Public transport in Sri Lanka is provided by the Ceylon Government Railways (C.G.R.) and the Ceylon Transport Board (C.T.B.) which operate the bus services. The C.T.B. carries 12 times as many passengers as the C.G.R. although the average passenger journey the former <sup>is</sup> less than one third of the latter. The C.T.B. operates a fleet of over 7,100 buses throughout the country and employs nearly 60,000 persons. The extent of the benefit to the lower income groups could be appreciated from the fact that the bus fares had remained at 3.75 cts. per mile for a period of 22 years<sup>2</sup> covering roughly 10 years of private ownership and 12 years of public ownership after the nationalisation in 1958. The fares were revised in 1971 the effect of which was to charge 20 cents for the first section

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<sup>1</sup> The only profession that remains unchanged is the "clerical and allied" category

<sup>2</sup> See Jones and Selvaratnam {1972 ; p. 158.}

and 5 cents per additional section. The net result is that a four mile journey today will cost about 30 cents or roughly 7.5 cents per mile, which is low by any standards.<sup>1</sup> Since the revision of fares the C.T.B has been either breaking even or operating at a small profit. But in 1975 a loss of about Rs.9.4 million was incurred and this should be regarded as a subsidy to the commuter. Likewise, the C.G.R. too operates its services at subsidised fares. In 1974, the C.G.R. incurred a loss of Rs.42.0 million and in 1975 a loss of Rs.49.2 million.

Apart from benefits to the commuters, cheap transport has also brought about social gains to the country as a whole. As pointed out by Karunatilake {1975}, it is because of the availability of cheap transport that large number of city workers travel from far off places rather than crowding in the city of Colombo or its suburbs.

There are a number of other goods and services provided at subsidised rates. For example Milk is one item which is subsidised to a level of about Rs.80.0 million per annum. Housing for the low income groups is another important service that is subsidised. Rent control laws have also

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<sup>1</sup> In comparison, the use of ones own car would cost about Rs.1.00 per mile and the use of a taxi about Rs.3.00 per mile.

offered relief to those in the lower middle, and middle income groups, dependent upon rental accommodation.

### 3.6 Agricultural Development

There is evidence to support the view that agricultural development primarily aimed at domestic production of essential food items has contributed a great deal towards the reduction of income disparities via increases in farmer incomes. We shall take note of the important steps taken in regard to agricultural policy, particularly some of the major legal enactments to which recent developments owe a great deal for providing the basic framework.

The Land Development Ordinance of 1935 could be regarded as the first major step taken towards increasing agricultural production during this century. This Ordinance introduced for the first time, the principle of Government initiative in alienating land and reflected an increasing concern for food production and for improving the position of the peasant cultivator (see Ellman, Ratnaweera et al. {1976} ). In each district, the Government Agent was in charge of selecting applicants and allocating Crown Land. In terms of this Ordinance, large numbers of local villagers as well as middle class Ceylonese were able to obtain Crown Land for purposes of cultivation. The next important legal enactment

was the Irrigation Ordinance of 1946, concerning the operation of irrigation facilities. This Ordinance spelt out clearly the division of responsibilities between the Irrigation Department and the cultivators: the Irrigation Department was responsible for maintaining the tanks and major channels and for controlling water-issues from the tanks, whilst the cultivators were required to pay an annual irrigation rate, and to maintain field channels, fences etc. This ordinance also included provision for summoning a meeting of all cultivators of a particular tract to be presided by the Government Agent for purposes of deciding upon the cultivation programme i.e. the extent of land to be sown, the varieties of paddy to be used, the time-table for planting, issue of water etc. (see Ellman, Ranaweera et al. {1976; p.19}). The cultivators were thus able to participate in the formulation of the programme although the government officials had the final say.

Although these two enactments provided the basic framework for agricultural development, yet there was no legal framework to safeguard the interests of the tenant cultivators (as opposed to owner cultivators) who formed the majority of cultivators. It is with this end in view that the Paddy Lands Act was introduced in 1958. It sought to increase the security of tenure of tenant cultivators of paddy lands and

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to regulate rents paid to landlords.

Whilst the above legal enactments provided the basic framework, a number of other measures were taken with a view to increasing agricultural production. The first important step taken after Independence was the introduction of a guaranteed price for paddy, fixed initially at Rs.8.00 per bushel. This was later changed to Rs.9.00 in 1950/51 to Rs.12.00 in 1951/52. It remained at the level till 1965 when it was changed to Rs.14.00 and later in 1973 to Rs.18.00. With effect from the latter part of 1974, the guaranteed price was revised to Rs.33.00. The next important step was that of introducing a Fertilizer Subsidy Scheme. A subsidy of as much as 50 per cent was given to cultivators who purchased fertilizer through the Department of Agrarian Services (see Karunatilake {1971;p.100}). As at the end of 1962 there were 42 fertilizer stores in the country. Two other important developments were the spread of agricultural credit, particularly after the establishment of the People's Bank in 1961, and the introduction of Crop Insurance on paddy from 1958-59 onwards. These efforts were further reinforced through the spread of agricultural extension services, intended for the diffusion of information regarding the sowing of high yielding varieties of paddy, fertilizer usage, improved agricultural techniques etc.

The results of these efforts are reflected in the steady increase of rice production, from a level of 31.3 million bushels of paddy in 1957 to the present level of about 85 million bushels. As shown by Karunatilake {1975 ; p.214}, import of rice as a percentage of total requirements had fallen steadily from a level of 60.75 per cent in 1950 to 21.08 per cent in 1974. Similar advances were made in regard to subsidiary food crops too. The total ban on the imports of potatoes in 1967 and restrictions imposed on the import of chillies, onions, pulses and other minor items gave adequate protection to the local producer.

From the standpoint of income distribution, the overall result would have been one of an income transfer to the producers of agricultural commodities, namely the farmer families. However, the overall impact on income inequality is not as clear cut as it seems since entrepreneurs in industry too would have benefited through the greater availability of foreign exchange (due to domestic food production) for import of raw materials.

The contribution to GDP from Agriculture, Forestry, Hunting and Fishing rose from Rs.2,846.0 million in 1963 to Rs.3,419.8 in 1973 (at constant 1959 prices),<sup>1</sup> representing a growth rate of 2.2 per cent per annum. Since there had been no significant

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<sup>1</sup> See Central Bank of Ceylon, Annual Report 1972, Appendix II Table 5 and Annual Report 1977, Appendix III, Table 2.

increases in the output of tea, rubber and coconut,<sup>1</sup> it follows that the major contribution to this growth rate of 2.2 per cent would have come from domestic agriculture, namely the cultivation of paddy and other subsidiary food crops. As reported by Jayawardena (see Chenery et al. {1974 p.275}) during period 1963-1969/70 paddy output had grown by 6.6 per cent, subsidiary crops by 6.5 per cent, small scale industry by 8 per cent, construction by 12 per cent and organised manufacturing by 7 per cent. These sectors had accounted for a little less than a third of GDP in 1970. The growth output of paddy and other subsidiary food crops may have contributed significantly towards the growth of per capita income in the rural sector. This increase in rural income would have contributed towards the reduction of the overall level of income inequality.

Agricultural production would have also contributed towards the reduction in income inequality within the rural sector itself (the Gini coefficient had declined from 0.424 in 1963 to 0.332 in 1973 - see Table 2.6 ). For, as argued by Jayawardena (see Chenery et al {1974 ; p.275}), paddy

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<sup>1</sup> Output of Tea had declined from 485 million lbs. in 1963 to 466 million lbs in 1973; output of Rubber had increased from 231 million lbs. in 1963 to 340 million lbs. in 1973: output of coconut had declined from 2,549 million nuts in 1963 to 1,935 million nuts in 1973. But the output of paddy increased from 49.2 million bushels in 1963 to 76.8 million bushels in 1974. (Source: Central Bank Reports)

holdings below 5 acres accounted for 95 per cent of the total number of holdings and 85 per cent of the cultivated area. Since paddy incomes accrue mostly to households in the income range of the bottom 40 per cent, as in the case of subsidiary crops and small industries, increases in the output of these commodities could have contributed towards the reduction of income inequality in the rural areas.

During the past five years there were several other developments, the effects of which are yet to be felt. The first was the Land Reform Law No.1 of 1972. This law fixed ceilings on the ownership of land on the basis of 25 acres per person in the case of paddy land and 50 acres per person in the case of other land. Under this law some 550,000 acres of land were taken over and vested in a number of institutions such as Divisional Land Reform Authorities (32 per cent), the Land Commission (20 per cent), USAWASANA (16 per cent), State Plantations Corporation (6 per cent) and Co-operative organisations. While this law covered only privately owned lands, the Land Reform (Amendment) Law of 1975 extended the principle ceilings on land ownership to estates owned by public companies as well, under this law, company estates of over 50 acres were taken over with compensation and vested in the state. The acreage taken over totalled about 415,000 acres of highly



productive tea, rubber and coconut lands. The largest part of this land taken over is now managed by JANAWASAMA, a state enterprise set up for the purpose of managing the estates.

Other important developments were the Agricultural Productivity Law No.2 of 1972 intended to ensure that the lands taken over are properly utilised and developed, the Agricultural Lands Law No.42 of 1973 which superceded the Paddy Lands Act, and the Sale of State Lands Law No.43 of 1973 to provide for the sale of state lands to individual cultivators and for the repeal of certain provisions of the Land Development Ordinance.

The impact of these changes could be assessed only during a future period.<sup>1</sup>

### 3.7 Summary

In this chapter, we have attempted to isolate the important factors that could have contributed towards the reduction of income inequality in recent years. There has been no clear

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<sup>1</sup> In an interesting discussion by Martha de Melo {1978; p. 183} it has been pointed out that Land reform would contribute positively towards reduction of income inequality but could lower the rate of capital formation (due to lower savings) and hence lower growth of GNP.

cut way of quantifying the contribution from each factor.

It is doubtful whether such an exercise could be done even at a future date because of the interrelationships between the factors involved. For example, better education apart from being a separate factor could influence better standards of health and sanitation.

. As could be seen from the variety of measures introduced, high priority had been assigned by the politicians and policy makers towards improving the welfare of the people in the short term. Although there had been at no stage any formal concept of a social welfare function, there appears to have been an underlying assumption that the welfare of the people depended upon better nutrition, health, education, minimum wages, transport facilities and a few other services. As noted previously, welfarism became an integral part of the political philosophy of major political parties and specific steps were taken by successive governments to provide these goods and services to the broad masses. The level of expenditure on welfare services have come under much criticism in recent years on account of the heavy burden placed upon the government budget and the concomitant constraints on resources for investment; public investment became overly dependent upon domestic and foreign borrowing. To the extent that investments are low, the production of goods and services in the future will be low (technology

remaining constant) and would inevitably lead to a diminution of future welfare. Thus the level of expenditure on welfare services has brought into focus the conflict between present and future welfare i.e present welfare versus growth.

The present climate<sup>1</sup> of thought has moved distinctly towards a programme of stable and self sustaining growth, the major objectives being growth of output and employment and an improvement in the balance of payments. The removal of rice subsidy from the richer half of the population, referred to earlier, could be seen as a concerted effort to mobilize resources for development; subsidies are now restricted to the target group consisting of households earning less than Rs.300 per month. In addition the dual exchange rate which had denied adequate incomes to tea and rubber producers has been abolished and a more realistic unified rate of exchange has been adopted and allowed to float in relation to other currencies. This change has made it possible to liberalise imports except for 139 scheduled items, and was intended to enable industrialists import adequate quantities of raw material and plant on the one hand and to satisfy consumer demand on the other.

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<sup>1</sup> See Government of Sri Lanka {1978} - The Medium Term Investment Programme 1979-83 (Mimeo), Ministry of Finance & Planning.

The Budget of November, 1977 also made proposals to reform the tax structure so as to provide greater incentives for production by granting five year tax holidays to new companies which engage<sup>in</sup> food production, horticulture, animal husbandry, off-shore and deep-sea fishing and to small and medium scale industrial ventures located outside Colombo. With a view to generating greater savings for investment, the deposit rates of one of the principal state banking institutions i.e the National Savings Bank were raised from 7-8 per cent to 12 to 18 per cent for 6 to 8 month deposits.

However current development plans have also laid emphasis on providing equal opportunities for the various segments of the community to benefit from economic growth. Accordingly, special measures to look after the old and the needy are to continue whilst health and education facilities are to continue to be available to all. In addition, the target groups are to be further protected from the harsh effects of inflation through appropriate pricing and wage policies. Thus, in broad terms the overall strategy of current development plans is to bring about a higher rate of economic growth without major sacrifices in welfare. It is in this context that we shall attempt to explore some of the issues concerning a strategy for redistribution with growth in the next chapter.

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CHAPTER 4A MODEL FOR REDISTRIBUTION WITH GROWTH

"Once upon a time the Kingdom of Solvia was gripped by a great debate. 'This is a growing economy but it can grow faster', many argued. 'Sustainable growth is best', came the reply, 'and that can come only from natural forces'"  
- E.S. Phelps (The Golden Rule of Accumulation: A Fable for Growthmen)

## 4.1 Introduction

The results of Chapter 2 have indicated that significant progress has been made in Sri Lanka in achieving a more equitable distribution of incomes during the recent past; the share of income accruing to the top ten per cent of the spending units had declined from 37 per cent in 1963 to 28 per cent in 1973, whilst the share of the bottom forty per cent rose from 14 per cent in 1963 to 19 per cent in 1973. Further, the Gini Coefficient in respect of spending units had declined from 0.45 in 1963 to 0.35 in 1973. As seen earlier, these results compare very favourably with patterns of income distribution prevailing in other countries. A more equitable distribution of incomes has been achieved in Sri Lanka through a number of policy measures, and these policy measures were reviewed in the previous chapter.

Having achieved this measure of success in the redistribution of incomes, a question may now be posed as to the nature and extent of emphasis that should be assigned to redistributive considerations vis-a-vis economic growth in future development plans. Let us first look at the background of growth rates achieved in the recent past. The rates of growth of GNP for each of the years during the period 1966-1977 can be seen from the table below:

Table 4.1

Growth rates of the Gross National Product  
at constant (1959) Factor Prices

<u>Year</u>	<u>Growth Rates</u>
1966	3.5 %
1967	5.0 %
1968	8.4 %
1969	4.5 %
1970	4.1 %
1971	0.4 %
1972	3.2 %
1973	3.8 %
1974	3.8 %
1975	2.8 %
1976	3.0 %
1977	4.4 %

Sources: Central Bank Annual Report 1975 - Table 1  
and Central Bank Annual Report 1977 -  
Table 1.

The growth rate of GNP for the above period averages to 3.9 per cent per annum. During the same period the country's population increased from 11.44 million in 1966 to 13.97 million in 1977, averaging to an increase of about 1.8 per cent per annum. Thus, the growth rate of per capita GNP averaged to 2.1 per annum. The growth rate of GNP witnessed during the latter part of this period has in fact been much lower than that planned for. The Five Year Plan (1972-76) envisaged a growth rate of 6 per cent per annum; but as seen from the above table, actual growth rates have only ranged from 2.8 per cent to 3.8 per cent during the period 1972 to 1975. Thus, the inescapable conclusion that one is led to is that although notable progress has been made in Sri Lanka in achieving a more equitable distribution of incomes, the growth of total income has been somewhat disappointing. But welfare depends upon absolute levels of income<sup>1</sup> as well and as noted earlier it is only through an increase of total income that there will be anything significant to redistribute. It would therefore appear that growth considerations should receive high priority if total income and thereby total welfare is to be increased. What would seem ideal is to regard further redistribution as an integral part of a growth cum redistribution process.

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<sup>1</sup> For example in a relatively simple case, welfare is a function of consumption which is in turn a function of income.

The aim of this chapter is firstly to explore by means of a simple model the interdependencies that could exist between the incomes of the rich and the poor. Thereafter, we shall explore as to how the respective income shares would behave as incomes grow. This would enable us to examine the impact of further redistributive measures upon income growth and to explore the possibilities of redistributive measures which will have the least possible repercussions upon income growth.

The relationship between economic growth and income distribution is bound to be a complex one. Although the 'causes' of economic growth such as savings and investment, technical progress, nutrition, health and education of the labour force, psychological, social and political factors etc., have been widely discussed in the literature, yet much attention does not seem to have been paid to the forces that shape the income distribution structure of a society. As noted in Chapter 1, discussions have mainly centered round the distribution of incomes between the factors of production namely capital and labour and not on the distribution of income between persons. Perhaps the first serious attempt to consider the latter was Kuznets' classic article of 1955. He differentiated between groups of forces that tend to increase inequality, namely the concentration of savings in the upper income brackets and the process of industrialization and urbanization and

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groups of forces that tend to counteract these such as legislation aimed at limiting the accumulation of property, the rise of more profitable industries and new entrepreneurs and the rise of service income in the lower brackets, accruing from professional and entrepreneurial pursuits. These and other factors such as the ownership of the factors of production, education, trade unionism, nationalization, pricing policies etc., do have a bearing upon the structure of income distribution in a country. The model developed in this chapter does not aim at incorporating all these factors; no model could possibly capture all the complexities of reality. Rather, the aim here is to take into account only a few relevant factors in an attempt to investigate the relationships that could exist between income distribution and economic growth. The emphasis is on gaining insight into the interdependencies between the growth of incomes of the rich and the poor, and the results of government participation in a growth cum redistribution programme. We could then examine as to how such insights could be of help in drawing appropriate policy conclusions. A point of particular interest would be to examine whether the model developed could explain or generate the behaviour pattern of income inequality with respect to income growth, hypothesised by Kuznets {1955; p.18} as follows:

"One might thus assume a long swing in the inequality characterizing this secular income structure: widening in the early phases of economic growth when the transition from the pre-industrial to the industrial civilization was most rapid; becoming stabilised for a while; and then narrowing in the later phases ... "

Paukert {1973} in setting out to test this hypothesis concluded that:

"The data presented in this article support the hypothesis expressed but not fully tested by Kuznets and Oshima that with economic development income inequality tends to increase, then become stable and then decrease. These data show clearly that there is an increase in inequality as countries progress from below \$100 level to the \$101-200 level and beyond. They establish that the peak of inequality is reached in the groups with a per capita income between \$200 and \$500."

An attempt was made to test the Kuznets hypothesis (see Codippily {1974} by means of a regression analysis of inter-country data using four relevant variables, namely level of per capita income, extent of school education, degree of dualism, and share of government and corporate savings in total savings. One of the conclusions reached was that there was a reasonable amount of evidence in support of the Kuznets hypothesis.

A study which at first sight appears to be at variance with the Kuznets hypothesis is the one carried out by Adelman and Morris {1971}. They examined the relationship between income distribution and 31 economic, socio-cultural and political factors and concluded that six factors which were most significant in explaining variations in income distribution were: (1) the rate of improvement of human resources, (2) abundance of natural resources, (3) extent of direct government economic activity, (4) extent of dualism, (5) potential for economic development and (6) extent of political participation. The level of per capita income did not emerge as a significant explanatory variable when taken together with all the other variables. However, it could be argued that the level of development represented by per capita income is in fact connected to varying extents with the six factors listed above (i.e. for instance rate of improvement of human resources will tend to be higher in richer countries etc.,) and that the Adelman and Morris study establishes the relationship between income distribution and the level of development not directly, but through several of its facets. Further, as argued by Paukert it would be possible for an important factor to be overshadowed by others if its impact is not linear. For these reasons it could be argued that the Adelman and Morris study is not at variance with the Kuznets hypothesis. The general evidence being in favour of the Kuznets hypothesis, as mentioned

earlier, a point of interest would be to examine the model vis-a-vis the Kuznets pattern.

There are other interesting directions in which the model could be developed. For instance, it would be possible to investigate as to how the model could be linked up with the incentive effect discussed by Codippily {1974}. In any redistribution programme, there could be incentive losses to the rich and incentive gains to the poor. One would therefore have to consider the net effect and its impact upon total income with a view to assessing the implications upon a growth cum redistribution programme. Another important exercise would be to investigate how considerations of optimality could be built into the model on the lines proposed by Hamada {1967}.

#### 4.2 General Features of the Model

The model developed here has its origins from the Chenery-Ahluwalia Model {1974: pp.209-235} and retains some of its general features. However, in many respects, it goes beyond the Chenery-Ahluwalia model. To begin with, the government will be introduced as an explicit sector participating in a growth cum redistribution process. Secondly, the type of redistribution of consumption discussed by Codippily {1974} will be incorporated into the model. Again, an attempt will be made to incorporate the incentive effect discussed by

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Codippily {1974}, but with further elaboration. The role of financial institutions, direct taxes, indirect taxes and foreign aid will be introduced. Further, investigations will be made as to the extent to which the model could be linked up with an optimal growth programme. We shall draw specific policy conclusions wherever possible and assess their effectiveness.

The methodology adopted in this study differs from that used in the Chenery-Ahluwalia study. The dynamic properties of the Chenery-Ahluwalia model are examined in their study in terms of simulation techniques rather than by analytical techniques. Although it is stated that their model can be written in the form of a set of difference equations, yet in anticipation of certain difficulties that would arise due to the presence of certain parameters that change over time, they had opted to use simulation techniques. In this study we shall neither use difference equations (in view of analytical difficulties that may arise) nor use simulation techniques. Instead, we shall formulate the model in terms of a set of simultaneous differential equations.

For the purpose of this model, all spending units<sup>1</sup> will be divided into two groups - the higher income group and the lower income group. As in the case of the Chenery-Ahluwalia model

<sup>1</sup> Although it may be preferable to talk in terms of households, yet from considerations of conformity with Chapter 2, we shall carry out the analysis in terms of spending units.

{1974; p.210} we shall regard as "rich" those spending units belonging to the top 20 per cent when spending units are ranked by income levels. The balance 80 per cent will be regarded as "poor". In terms of the CFS 1973, the latter category would approximately consist of those who received incomes below Rs.400 per month in 1973. The government is considered to be an 'impersonal' agency which effects redistributive policies and participates in a growth cum redistribution process. The government is termed 'impersonal' since it is assumed that total welfare is a function of only the utilities of the rich and the poor. The utility of the government as such is not assigned any meaning and does not enter into welfare considerations.

Admittedly it would have been more realistic to formulate the model in terms of several income groups, say four or five or the number of income groups used in Chapter 2. Further, one could take into account differences as between urban, rural and estate households, and regional differences. But the consequences of such an approach will be one of increasing complexity and a number of analytical results which will be intractable from the point of view of gaining insight and drawing policy conclusions. Given the deficiencies in the information system and the rather shaky data base we are starting with, the emphasis will be more on deriving directions for action

rather than precise magnitudes and targets.

Following the Chenery-Ahluwalia approach, we shall assume a dualistic pattern of production in which there are capitalistic modes of production based on the use of hired labour side by side with traditional modes of production based on self employment and family labour. Capital owned by the rich is assumed to be of two types - that which generates income with the use of hired labour, termed 'linked capital' and that which generates income without use of hired labour, termed 'non linked capital'. As in the Chenery-Ahluwalia model, we may think of this form of capital as a type which uses only highly skilled labour or professionals belonging to the higher income group or capital invested abroad. The meaning of this form of capital will be enlarged when we consider financial institutions later on. Capital owned by the poor is assumed to be only of one type and that it is used only for self employment.

The production equations used initially are assumed to be of the simplest type i.e. that where output (value added) is regarded as a function of capital only, and based upon fixed capital-output ratios. As often argued, the justification for this lies in the overall scarcity of capital in the developing countries (in contrast to the abundance of labour) as a factor of production. But skilled manpower is also a scarce factor,

and for this reason, a modified production function will be considered when we discuss the incentive effect. Wage and profitability parameters are assumed to be different for the two income groups. With regard to savings behaviour, a fixed savings rate is assumed for the rich (which could be relaxed if necessary) and a progressively rising savings rate for the poor in accordance with empirical observation: that is, a fixed marginal propensity to save is assumed in respect of the poor. As in the case of the Chenery-Ahluwalia model, growth theory made use of here is of the Harrod type, and applied to the individual sectors. For purposes of completeness, we may note that the basic assumption in Harrod's approach are as follows:

- (a) the level of income is the most important determinant of the supply of savings;
- (b) the rate of increase of income is an important determinant of the demand for savings;
- (c) demand for savings equals the supply of savings.

In addition it is assumed that we are in a 'one good' world, and that the capital-output ratios are constant over the time span considered. Depreciation of the total capital stock is ignored...

The above are the general features of the model. Special features will be described wherever variations are discussed.

#### 4.3 The Model

The distribution of capital is assumed to be as follows:



$K_1^l$  = capital owned by the rich using hired labour;

$K_1^n$  = capital owned by the rich not using hired labour;

$K_2^n$  = capital owned by the poor for self employment;

$K_g^l$  = capital owned by government using hired labour.

Following the Chenery-Ahluwalia model we may regard  $K_1^n$  as capital which employs only highly skilled labour from the higher income group or as capital invested abroad or both. When financial institutions are introduced subsequently, the interpretation of  $K_1^n$  could be enlarged.

The production functions are assumed to be of the Harrod type initially, and as mentioned previously the capital-output ratios are held constant, during the time span considered. Then, the output generated by these stocks of capital are given by the production equations:

$$Q_1^l = a_1 K_1^l$$

$$Q_1^n = b_1 K_1^n$$

$$Q_2^n = b_2 K_2^n$$

$$Q_g^l = a_g K_g^l \quad \dots \quad (4.1)$$

where the  $Q$  s denote the respective outputs and the  $a$  s and the  $b$  s are the corresponding output-capital ratios. The subscript 1 will denote the higher income group, subscript 2 the lower income group and subscript  $g$  the government.

The outputs (value added) so generated will be divided between wages and profits. By virtue of employment in the modern sector both the rich and the poor will receive shares of the output  $Q_1^l$  as wage incomes. Likewise, both the rich and poor will receive wage incomes from employment in government.

$$\begin{array}{llll} \text{Thus, wage income of the rich } W_1 = w_{11} Q_1^l + w_{1g} Q_g^l \\ \text{" " " " poor } W_2 = w_{21} Q_1^l + w_{2g} Q_g^l \dots (4.2) \end{array}$$

where the  $w$  s are the wage parameters. In other words  $w_{11}$  is the proportion of  $Q_1^l$  accruing to the rich,  $w_{21}$  the proportion of  $Q_1^l$  accruing to the poor and so on.

Likewise, the profit incomes accruing to the rich, poor and government can be expressed as:

$$P_1 = p_1 Q_1^l + Q_1^n$$

$$P_2 = Q_2^n$$

$$P_g = p_g Q_g^l \dots (4.3)$$

where the  $p$  s are the profitability parameters.

It should be noted that wage and profitability parameters are not independent of each other as they represent shares of wages and profits from total output. Accordingly, they should add up so as

to yield total output, which implies that

$$\begin{aligned} w_{11} + w_{21} + p_1 &= 1 \\ \text{and } w_{1g} + w_{2g} + p_g &= 1 \end{aligned} \quad \dots \quad (4.4)$$

Adding equations (4.2) respectively to the first two equations of (4.3) and writing the third equation of (4.3) separately, the total incomes of the rich, poor and government are thus defined by

$$\begin{aligned} Y_1 &= w_{11} Q_1^l + w_{1g} Q_g^l + p_1 Q_1^n + Q_1^n \\ Y_2 &= w_{21} Q_1^l + w_{2g} Q_g^l + Q_2^n \\ Y_g &= p_g Q_g^l \end{aligned} \quad \dots \quad (4.6)$$

Equations (4.1) may now be made use of to eliminate the  $Q$  s and to yield the following equations:

$$\begin{aligned} Y_1 &= (w_{11} + p_1) a_1 K_1^l + b_1 K_1^n + w_{1g} a_g K_g^l \\ Y_2 &= w_{21} a_1 K_1^l + b_2 K_2^n + w_{2g} a_g K_g^l \\ Y_g &= p_g a_g K_g^l \end{aligned} \quad \dots \quad (4.7)$$

These equations could also be written in the matrix form as follows

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_g \end{bmatrix} = \begin{bmatrix} (w_{11} + p_1) a_1 & b_1 & 0 & w_{1g} a_g \\ w_{21} a_1 & 0 & b_2 & w_{2g} a_g \\ 0 & 0 & 0 & p_g a_g \end{bmatrix} \begin{bmatrix} K_1^l \\ K_1^n \\ K_2^n \\ K_g^l \end{bmatrix} \quad \dots \quad (4.8)$$

Thus the distribution of incomes is determined by the distribution of capital stocks and the wage and profitability parameters. Or, in the language of matrix algebra, the vector of incomes is determined by the product of the wage and profitability matrix and the vector of capital stocks.

#### 4.4 Static Considerations

The picture presented by equations (4.7) is essentially a static one. Nevertheless, there are certain policy implications that could be deduced from these equations, subject to assumptions of validity (to be discussed later). For example, we could pose the question: what sort of capital stock should be expanded in order to produce the greatest impact on the incomes of the poor? Subject to the simplifying assumptions we have made, we could seek an answer to this question by looking at the second equation of (4.7) and examining as to which type of capital when expanded by a unit amount will bring about the greatest impact upon the income of the poor represented by  $Y_2$ . The answer would obviously depend upon the relative magnitude of the coefficients  $w_{21} a_1$ ,  $b_2$  and  $w_{2g} a_g$ . The coefficient  $b_2$  is the output-capital ratio in self employment and  $a_1$  is the output-capital ratio in the modern sector. The coefficient  $w_{21} < 1$  by virtue of the definition of  $w_{21}$  as the proportion of wages of the poor from the output  $Q_1^1$

$$\text{Suppose } b_2 > w_{21} a_1 \quad \dots \quad (4.9)$$

<sup>1</sup> In fact  $b_2 > w_{21} a_1$  in the numerical examples considered later.

Further, it would seem reasonable to assume that the output-capital ratio  $a_g$  in the government sector is lower than  $b_2$  in self employment. Even if it were not so, there is a strong possibility that  $b_2 > w_{2g} a_g$  by virtue of  $w_{2g}$  being less than one. Thus, so long as

$$\begin{aligned} b_2 &> w_{21} a_1 \\ \text{and } b_2 &> w_{2g} a_g \end{aligned} \quad \dots \quad (4.10)$$

it would follow that a unit expansion in  $K_2^n$  will have a greater impact on the incomes of the poor than unit expansions in either  $K_1^l$  or  $K_g^l$ . That is, from the standpoint of improving the incomes of the poor, policies which stimulate an increase of capital for self employment appear to be preferable to those that expand state capital. Capital for self employment could be owned either in co-operative form or individually.

Another aspect we could discuss on the basis of equations (4.7) concerns nationalisation. Nationalisation involves transfer of capital assets from the private sector to the government sector. Accordingly, let us assume that capital assets worth  $\Delta K_1^l$  is transferred from a stock of capital  $K_1^l$  owned by the rich to the stock of capital owned by the government, namely  $K_g^l$ . Then, from the second equation of (4.7), it follows that the change in incomes of the poor is given by

$$\Delta Y_2 = (w_{2g} a_g - w_{21} a_1) \Delta K_1^l \quad \dots \quad (4.11)$$

The change will be a positive one, i.e there will be an increase

in the incomes of the poor as long as

$$w_{2g} a_g > w_{21} a_1 \quad \dots (4.12)$$

Let us also assume that compensation paid is also of value  $\Delta K_1^l$ . Then, if the rich decide to use this amount in increasing their stock of non linked capital, it follows from the first of equations (4.7) that the change in their incomes will be

$$\Delta Y_1 = [b_1 + w_{1g} a_g - (w_{11} + p_1) a_1] \Delta K_1^l \quad \dots (4.13)$$

Thus, the incomes of the rich can increase or remain unchanged or decrease according as

$$b_1 + w_{1g} a_g \gtrless (w_{11} + p_1) a_1 \quad \dots (4.14)$$

Admittedly, the above arguments have been carried out in an aggregative form. For instance, we have assumed that the parameters and the output-capital ratios relating to aggregate output hold good from activity generated by  $\Delta K_1^l$ . If there are appreciable differences in the parameters, then suitable adjustments have to be made by replacing them with estimates relevant to the output generated by  $\Delta K_1^l$ . Again caution is necessary in interpreting the inequality (4.12). Although  $a_g$  is generally less than  $a_1$ , yet, since  $w_{2g}$  could be much greater than  $w_{21}$ , inequality (4.12) could hold. The coefficient  $w_{2g}$  being much greater than  $w_{21}$  does not imply that workers of a nationalised

venture will receive an increase in wages soon after nationalisation. On the contrary, wage rates may be lower or at best remain the same. But a greater number is likely to be employed. It is the share of wages of the workers including the new recruits that could be higher than before nationalisation. Thus a positive impact upon the incomes of the poor could take place through employment.

A question which is difficult to answer in terms of the preceding analysis is whether income inequality will increase or decrease after nationalisation. If inequality (4.12) holds and the incomes of the rich decrease according to (4.14), then obviously, income inequality will decrease. But according to inequalities (4.12) and (4.14), it is possible that incomes of both the rich and the poor could increase. In such a situation precise magnitudes are necessary in order to assess whether income inequality has increased or decreased. But how could both the incomes of the rich and the poor increase simultaneously? Such a possibility could take place if, after nationalisation, the share of profits decrease and the share of wages increase, which implies a decrease in government income.

Before leaving the topic of nationalisation, it should be noted that a complete discussion cannot be carried out in terms of a simple model. The decision to nationalise a particular venture

must necessarily be analysed in terms of the set of circumstances peculiar to it and the related social benefits and costs.

#### 4.5 Dynamic Considerations

The preceding discussions have resulted essentially from static considerations of the model. What would seem more important is to consider distributional questions over time. In order to do this the model must be developed into a dynamic one. The essential elements for this are the role of savings and capital accumulation which are defined by the equations below. As mentioned earlier, growth in each sector is assumed to be of the Harrod type. Equating investment to savings we get

$$\begin{aligned}\dot{K}_1^l &= q s_1 Y_1, \quad 0 < q < 1 \\ \dot{K}_1^n &= (1 - q)s_1 Y_1 \\ \dot{K}_2^n &= s_0 + s_2 Y_2 \\ \dot{K}_g^l &= s_g Y_g \quad \dots \quad (4.15)\end{aligned}$$

where  $s_1$  = average (and marginal) propensity to save of the rich,  
 $s_2$  = marginal propensity to save of the poor,  
 $s_0$  = constant term in savings function of the poor,  
 $s_g$  = average (and marginal) propensity to save of,  
     g government  
 $q$  = proportion of savings of the rich diverted to  
     increase of  $K_1^l$



It is further assumed that the average propensity to save of the poor does not exceed the average propensity to save of the rich. As in the Chenery-Ahluwalia model, it is assumed that once the income of the poor (in per capita terms) reaches the initial per capita income level of the rich (i.e. income at time  $t = 0$ ), then the savings function of the poor will be the same as that of the rich. Such an assumption is necessary so as to ensure that within the framework of the model, the savings rate of the poor does not at any time exceed the savings rate of the rich.

To summarise, the main features of the model to be discussed are defined by the following equations, subject to the assumptions made in this section-

$$\begin{aligned} Y_1 &= (w_{11} + p_1) a_1 K_1^l + b_1 K_1^n + w_{1g} a_g K_g^l \\ Y_2 &= w_{21} a_1 K_1^l + b_2 K_2^n + w_{2g} a_g K_g^l \\ Y_g &= p_g a_g K_g^l \end{aligned} \quad \dots (4.7)$$

$$\begin{aligned} \dot{K}_1^l &= q s_1 Y_1 \\ \dot{K}_1^n &= (1-q) s_1 Y_1 \\ \dot{K}_2^n &= s_0 + s_2 Y_2 \\ \dot{K}_g^l &= s_g Y_g \end{aligned} \quad \dots (4.15)$$

From the above equations, the  $\dot{K}$  s could be eliminated to yield

a set of linear differential equations in the  $Y$ 's so as to define the time paths of  $Y_1$ ,  $Y_2$ ,  $Y_g$ . But before we do this, it is relevant to compare these equations with some of the results of the Kaldor-Pasinetti<sup>1</sup> model. For purposes of comparison, we should set  $Y_g = 0$ . Then, by addition of equations (4.15) we get

$$\begin{aligned}
 \text{Investment } I &= \dot{K}_1^l + \dot{K}_1^n + \dot{K}_2^n = s_1 Y_1 + s_0 + s_2 Y_2 \\
 &= s_1 Y_1 + s_0 + s_2 (Y - Y_1), \text{ (Total income } Y = Y_1 + Y_2) \\
 &= (s_1 - s_2) Y_1 + s_0 + s_2 Y \\
 \text{Therefore, } \frac{I}{Y} &= (s_1 - s_2) \frac{Y_1}{Y} + \frac{s_0}{Y} + s_2 \\
 \text{and } \frac{Y_1}{Y} &= \frac{s_2}{s_2 - s_1} + \frac{s_0}{(s_2 - s_1)Y} - \frac{1}{(s_2 - s_1)} \frac{I}{Y} \quad (4.16)
 \end{aligned}$$

This equation defines the distribution of income between the rich and the poor. Equation (4.16) is the analogue<sup>2</sup> of equation (8) in Pasinetti {1961} which is

$$\frac{P_c}{Y} = \left[ \frac{1}{s_c - s_w} \right] \frac{I}{Y} - \frac{s_w}{s_c - s_w} \quad (4.17)$$

where  $P_c$  = income of capitalists,

<sup>1</sup> Refers to Kaldor's model of 1955 as corrected by Pasinetti later in 1961.

<sup>2</sup> If we set  $s_0 = 0$ , then equation (4.16) becomes

$$\frac{Y_1}{Y} = \frac{1}{s_1 - s_2} \frac{I}{Y} - \frac{s_2}{s_1 - s_2}$$

which is an exact analogue of the Pasinetti equation.

$S_c$  = average (and marginal) propensity to save of capitalists,

$S_w$  = average (and marginal) propensity to save of workers.

Equation (4.16) is slightly more general than the Pasinetti results since we have assumed that the rich derive income not only from profits but also from wages, and that the savings function of the poor is not a proportional savings function but of the form specified in equation (4.15). As in Pasinetti's equation, the importance of the propensity to save of the poor in regard to the distribution of incomes is clearly demonstrated in equation (4.16).

The Kaldor-Pasinetti approach also discusses the factors influencing the distribution of incomes between profits and wages. However, they did not proceed to examine as to how the shares of income vary over time. The Chenery-Ahluwalia model and this study attempt to examine distribution questions over time and in a sense may be considered to be extensions of the Kaldor-Pasinetti approach.

#### 4.6 Growth without further redistributive measures

The object of this section is to examine the time paths of the incomes  $Y_1$ ,  $Y_2$ , and  $Y_G$  as well as the relative income shares. The underlying assumption is that redistributive

measures have been and are operative: thus the  $Y$ 's denote income shares that have resulted after the operation of the redistributive measures already introduced. But, it is also assumed that no further redistributive measures are introduced.

We begin by differentiating the equations (4.7) to get

$$\begin{aligned}\dot{Y}_1 &= (w_{11} + p_1) a_1 \dot{K}_1^l + b_1 \dot{K}_1^n + w_{1g} a_g \dot{K}_g^l \\ \dot{Y}_2 &= w_{21} a_1 \dot{K}_1^l + b_2 \dot{K}_2^n + w_{2g} a_g \dot{K}_g^l \\ \dot{Y}_g &= p_g a_g \dot{K}_g^l \quad \dots (4.18)\end{aligned}$$

Eliminating  $\dot{K}_1^l$ ,  $\dot{K}_1^n$  etc., using equations (4.15) we get

$$\begin{aligned}\dot{Y}_1 &= [(w_{11} + p_1) a_1 q + (1 - q) b_1] s_1 Y_1 + w_{1g} a_g s_g Y_g \\ \dot{Y}_2 &= w_{21} a_1 q s_1 Y_1 + b_2 s_0 + b_2 s_2 Y_2 + w_{2g} a_g s_g Y_g \\ \dot{Y}_g &= p_g a_g s_g Y_g \quad \dots (4.19)\end{aligned}$$

$$\text{i.e. } \dot{Y}_1 = \alpha_1 Y_1 + \gamma_1 Y_g$$

$$\dot{Y}_2 = \beta_0 + \alpha_2 Y_1 + \beta_2 Y_2 + \gamma_2 Y_g$$

$$\dot{Y}_g = \gamma_3 Y_g \quad \dots (4.20)$$

where

$$\begin{aligned}
 \alpha_1 &= [(w_{11} + p_1) a_1 q + b_1 (1 - q)] s_1 \\
 \gamma_1 &= w_{1g} a_g s_g \\
 \alpha_2 &= w_{21} a_1 q s_1 \\
 \beta_0 &= b_2 s_0 \\
 \beta_2 &= b_2 s_2 \\
 \gamma_2 &= w_{2g} a_g s_g \\
 \gamma_3 &= p_g a_g s_g \quad \dots (4.21)
 \end{aligned}$$

Equations (4.20) is the set of linear simultaneous differential equations which represent the manner in which  $Y_1$ ,  $Y_2$ ,  $Y_g$  change over time; their solutions will represent the time paths of  $Y_1$ ,  $Y_2$  and  $Y_g$ . But before proceeding to solve these equations, there are some interesting observations that could be made. Let us take the second equation of (4.20), divide throughout by  $Y_2$  and write it as

$$\frac{\dot{Y}_2}{Y_2} = \frac{\beta_0}{Y_2} + \beta_2 + \alpha_2 \frac{Y_1}{Y_2} + \gamma_2 \frac{Y_g}{Y_2} \quad \dots (4.22)$$

In the first instance, it is clear that the rate of growth of  $Y_2$  directly depends upon  $\beta_2$  i.e.  $b_2 s_2$  indicating the importance of the marginal savings rates of the poor and the output per unit of capital which could be realised by their capital stock, in

the growth of their incomes. Given the other interdependencies assumed, it is obvious that the rate of growth of incomes of the poor depend upon the other variables as well. But the above equation attempts to spell out the nature of these interdependencies. For example, the last term of equation (4.22) indicates that higher the government income will be in relation to the income of the poor  $Y_2$ , higher will be the growth rate of the income of the poor. But the impact of this term is diminished by the coefficient  $\gamma_2$  i.e.  $w_{2g} a_g s_g$ , which, as would be seen later is much less than one. The underlying reason for the positive contribution made by this term can be traced to the wage income of the poor received from government activity; to the extent that government income increases, there would be an increase in wage income as well. Similarly, the third term  $\alpha_2 \frac{Y_1}{Y_2}$  indicates that there is a positive contribution to the growth rate of  $Y_2$  from the income of the rich. Once again this contribution can be traced to the wage incomes received by the poor, by virtue of employment in establishments owned by the rich. The implication of this relationship is that policy measures aimed at reducing  $Y_1$ , without increases in  $Y_2$  or  $Y_g$  (say for example withdrawal of certain incentives - to be discussed later), will adversely affect the growth of incomes of the poor. Such adverse effects could take the form of either 'holding

down' of wages or even lay offs.

The above arguments should not be taken to imply that  $\frac{Y_1}{Y_2}$  should be increased in order to increase the growth rate of the income received by the poor. On the contrary, redistributive considerations would prompt a higher  $Y_2$  in relation to  $Y_1$ , i.e. a lower  $\frac{Y_1}{Y_2}$ . One must therefore examine the net effect on the growth rate  $\frac{\dot{Y}_2}{Y_2}$  arising out of the positive contribution due to an increase in  $Y_1$  and a negative contribution resulting from a lowering of  $\frac{Y_1}{Y_2}$ . In order to discuss such questions, the differential equations have to be solved in the first instance. Thereafter, we could trace the behaviour pattern of  $\frac{Y_1}{Y_2}$  over time and examine the consequence of further redistributive measures vis-a-vis growth of incomes.

Equations (4.20) is a set of linear simultaneous differential equations, the solution to which could be easily found as (see Appendix IV for solution)

$$Y_1 = \left[ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_E}{\gamma_3 - \alpha_1} \right] e^{\alpha_1 t} + \frac{\gamma_1 \bar{Y}_E e^{\gamma_3 t}}{\gamma_3 - \alpha_1}$$

$$Y_2 = \frac{-\beta_0}{\beta_2} + \left[ \bar{Y}_2 + \frac{\beta_0}{\beta_2} - \frac{\alpha_2}{\alpha_1 - \beta_2} \left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_E}{\gamma_3 - \alpha_1} \right\} - \frac{\bar{Y}_E}{\gamma_3 - \beta_2} \left\{ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right\} \right] e^{\beta_2 t}$$

$$+ \frac{\alpha_2}{\alpha_1 - \beta_2} \left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right\} e^{\alpha_1 t} + \frac{\bar{Y}_g}{\gamma_3 - \beta_2} \left\{ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right\} e^{\gamma_3 t}$$

$$Y_g = \bar{Y}_g e^{\gamma_3 t} \quad \dots (4.23)$$

The above equations represent the time paths of  $Y_1$ ,  $Y_2$  and  $Y_g$ .

Given the interdependencies assumed in the very formulation of the model, it is obvious that the growth of income of one group will depend upon that of the other. But the above solution shows in detail the nature of the interdependencies. For example the first equation in (4.22) can be written as

$$Y_1 = \bar{Y}_1 e^{\alpha_1 t} + \gamma_1 Y_g \frac{(e^{\gamma_3 t} - e^{\alpha_1 t})}{\gamma_3 - \alpha_1} \quad \dots (4.24)$$

Thus, the income of the rich at a given point of time depends not only upon the parameter  $\alpha_1$  defined by the first equation in (4.21) but also upon the rate of growth of government income  $\gamma_3$ , the exact functional form being shown by the above equation. An interesting feature is that irrespective of whether  $\gamma_3$  is greater or less than  $\alpha_1$  the second term of equation (4.24) is always positive indicating a positive contribution to  $Y_1$  from the growth of government income. Suppose  $\gamma_3 = 0$  (i.e. no growth of government income), then the second term reduces to

$$\gamma_1 \bar{Y}_g \frac{e^{\alpha_1 t}}{\alpha_1}$$

which is positive. The explanation for the positive contribution



could be traced to the wage income of the rich received from the government a part of which is saved and invested so as to increase the total income of the rich at a growth rate of  $\alpha_1$ .

The solution (4.23) exists even when  $\alpha_1 \rightarrow \gamma_3$ ; it is not undefined due to the presence of  $(\gamma_3 - \alpha_1)$  in the denominator of the second term. It could be shown that the second term exists and is positive, as follows:

$$\text{Let } Z_1 = \gamma_1 \bar{Y}_g \left( \frac{e^{\alpha_1 t} - e^{\gamma_3 t}}{(\alpha_1 - \gamma_3)} \right) \quad \dots (4.25)$$

Let  $\alpha_1 = \gamma_3 + \delta$ , where  $\delta$  is a small positive quantity.

$$\begin{aligned} \text{Then } \lim_{\alpha_1 \rightarrow \gamma_3} Z_1 &= \lim_{\delta \rightarrow 0} Z_1 = \lim_{\delta \rightarrow 0} \gamma_1 \bar{Y}_g \left\{ \frac{e^{(\gamma_3 + \delta)t} - e^{\gamma_3 t}}{\delta} \right\} / \delta \\ &= \gamma_1 \bar{Y}_g e^{\gamma_3 t} \lim_{\delta \rightarrow 0} \left( \frac{e^{\delta t} - 1}{\delta} \right) \end{aligned}$$

$$\begin{aligned} \text{Using the Mean Value Theorem, } \lim_{\delta \rightarrow 0} Z_1 &= \gamma_1 \bar{Y}_g e^{\gamma_3 t} \lim_{\delta \rightarrow 0} \left( \frac{\delta t e^{\eta t}}{\delta} \right), \quad 0 < \eta < \delta \\ &= \gamma_1 \bar{Y}_g e^{\gamma_3 t} \lim_{\delta \rightarrow 0} (t e^{\eta t}), \quad 0 < \eta < \delta \\ &= \gamma_1 \bar{Y}_g e^{\gamma_3 t} t \quad (4.26) \end{aligned}$$

Thus, when  $\alpha_1$  approaches  $\gamma_3$ , the solution (4.24) exists and the second term on the right assumes a limiting value of  $t \gamma_1 \bar{Y}_g e^{\gamma_3 t}$  which is positive. Alternatively, in solving the differential equation in  $Y_1$ , with  $\alpha_1 = \gamma_3$  it could be shown that  $t \gamma_1 \bar{Y}_g e^{\gamma_3 t}$

is the particular integral (see note at the end of Appendix IV).

The above result has no special significance. For, it merely means that if  $\alpha_1$  which is the main parameter associated with the growth of incomes of the rich ( $Y_1$ ) is equal to  $\gamma_3$  which is the rate of growth of government income, there will be a positive contribution to the growth of  $Y_1$  from the growth of government income.

As in the preceding analysis, we could re-arrange terms and write the second equation in (4.23) as:

$$\begin{aligned}
 Y_2 = \bar{Y}_2 e^{\beta_2 t} &+ \frac{\beta_0}{\beta_2} (e^{\beta_2 t} - 1) + \frac{\alpha_2}{\alpha_1 - \beta_2} \left\{ \bar{Y}_1 \right. \\
 &- \left. \frac{\gamma_1 \bar{Y}_r}{\gamma_3 - \alpha_1} \right\} (e^{\alpha_1 t} - e^{\beta_2 t}) \\
 &+ \frac{\bar{Y}_r}{\gamma_3 - \beta_2} \left\{ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right\} (e^{\gamma_3 t} - e^{\beta_2 t}) \quad \dots (4.27)
 \end{aligned}$$

It is clear from the above equation that the growth of incomes of the poor depends in the first instance upon the parameter  $\beta_2$  which as defined in equations (4.21) is the product  $b_2 s_2$  where  $b_2$  is the output-capital ratio and  $s_2$  is the marginal savings rate of the poor. This signifies the role of self reliance

namely that the poor will have to save more and invest in order to generate greater output.

Secondly, equation (4.27) shows that the growth of incomes of the poor also depends on the parameters  $\alpha_1$  and  $\gamma_3$ . As in the case of the preceding analysis, it could be shown that the third term in the above equation is positive<sup>1</sup> whether or not  $\alpha_1$  is greater than  $\beta_2$ ; the same applies to the fourth term of the above equation. Thus, there are positive contributions accruing to the income of the poor from the growth of government income and income of the rich. Once again, these positive contributions could be traced to the wage increases received by the poor by virtue of employment in the private sector and in government. Although these contributions are positive, a relevant question to ask at this stage is what would happen to  $Y_2$  if say  $\alpha_1$  is reduced? In order to examine this question a first approximation has to be made, namely to ignore  $\frac{\gamma_1 \bar{Y}_E}{\gamma_3 - \alpha_1}$  (see footnote)

$$\text{and } \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} \quad 2$$

---

<sup>1</sup> It would appear from the subsequent numerical examples that  $\bar{Y}_1$  is the dominant term in  $\left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_E}{\gamma_3 - \alpha_1} \right\}$

by virtue of which it is positive. Further, it is clear that

$$\frac{e^{\alpha_1 t} - e^{\beta_2 t}}{\alpha_1 - \beta_2} > 0$$

<sup>2</sup> Subsequent numerical examples will show that this term is much smaller than  $\gamma_2$ .

Subject to this approximation, the third term reduces to

$$\frac{\alpha_2 \bar{Y}_1}{\alpha_1 - \beta_2} (e^{\alpha_1 t} - e^{\beta_2 t})$$

Let us write

$$Z = \frac{e^{\alpha_1 t} - e^{\beta_2 t}}{\alpha_1 - \beta_2} \dots (4.28)$$

At any point of time  $t$ , the behaviour of  $Z$  in relation to the changes in  $\alpha_1$  has to be determined in terms of the partial derivative

$$\begin{aligned} \frac{\partial Z}{\partial \alpha_1} &= \frac{[(\alpha_1 - \beta_2)t - 1] e^{\alpha_1 t} + e^{\beta_2 t}}{(\alpha_1 - \beta_2)^2} \\ &= e^{\beta_2 t} \left[ \frac{\{(\alpha_1 - \beta_2)t - 1\} e^{(\alpha_1 - \beta_2)t} + 1}{(\alpha_1 - \beta_2)^2} \right] \dots (4.29) \end{aligned}$$

Clearly,  $e^{\beta_2 t} > 0$  and  $(\alpha_1 - \beta_2)^2 > 0$ . Therefore the sign of  $\frac{\partial Z}{\partial \alpha_1}$  would depend upon the sign of the expression within square brackets. We may write this expression as

$$V = (ht - 1)e^{ht} + 1 \dots (4.30)$$

$$\text{where } h = \alpha_1 - \beta_2$$

It could be shown that  $V \geq 0$  for all  $ht$  as follows:

Let us denote  $ht$  by  $x$  for convenience and write (4.30) as

$$V = e^x (x - 1) + 1 \dots (4.30)$$

When  $x = 0$ ,  $V = 0$ . Therefore curve passes through origin.

$$\frac{dV}{dx} = e^x + e^x (x - 1) = xe^x = 0 \text{ when } x = 0, \text{ and}$$

there is a turning point at  $x = 0$

$$\frac{d^2 V}{dx^2} = xe^x + e^x = 1 \quad \text{when } x = 0$$

∴ The turning point at  $x = 0$  is a minimum point.

$V \geq 0$  for all  $x$  i.e. for all  $ht$ .

#### Behavior of $V$

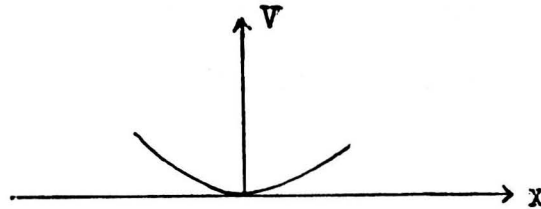


Figure 4.1

Alternatively, this result follows from a first approximation of (4.30) For,

$$\begin{aligned} V &\approx (ht - 1)(1 + ht) + 1 \\ &= (ht)^2 - 1 + 1 = (ht)^2 \geq 0 \end{aligned} \quad \dots (4.31)$$

It should be noted that this result holds good irrespective of whether  $ht > 0$  or not i.e. irrespective of whether  $\alpha_1$  is greater or less than  $\beta_2$ . Going back to equation (4.29) it follows that

$$\frac{\partial Z}{\partial \alpha_1} > 0 \text{ and consequently that } \frac{\partial Y_2}{\partial \alpha_1} > 0 .$$

Thus an attempt to lower  $\alpha_1$  will result in lowering  $Y_2$ . The parameter  $\alpha_1$  is the main parameter associated with the growth of incomes of the rich and is defined in terms of several other parameters as shown in equations (4.21). For example

a lowering<sub>Λ</sub><sup>α<sub>1</sub></sup> of  $\alpha_1$  could take place through a lowering of  $s_1$  or  $w_{11}$  or  $p_1$ . The policy implication of this result is that measures designed to curb the growth of incomes of the rich will have a negative impact upon the income of the poor. Since the poor receive an employment income from the rich, the negative impact could take the form of slowing down expansion of employment or even lay offs if  $\alpha_1$  is sufficiently lowered.

#### 4.7 Tests for Validity of the Model

As mentioned earlier, the derivation of policy implications and related discussions have been carried out subject to assumptions of the validity of the model. The validity of a model depends upon the extent to which it can explain empirical reality. In the model developed herein, we have taken into account some of the relevant factors. Accordingly, the validity of the model would depend upon obtaining some form of explanation of empirical observations. One empirical result with which the model could be tested is the Kuznets curve discussed earlier, according to which as a low income country develops, inequality of income tends to increase at first, reach a maximum and then decrease. In other words, income inequality should trace an inverted "U" shape over time. We should investigate as to whether the model developed can in general trace such a path - in other words, whether in general terms such a pattern is implicit in the model.

Since we have considered only two income groups, we may use the ratio  $\frac{Y_1}{Y_2}$  as a measure of inequality and investigate the behaviour of this ratio over time. The behaviour pattern can be analysed by means of numerical examples. But before we do so, we could examine the behaviour pattern in analytical terms but subject to certain first approximations.

The solution (4.23) can be written as

$$\begin{aligned} Y_1 &= A_1 e^{\alpha_1 t} + C_1 e^{\gamma_3 t} \\ Y_2 &= A_2 e^{\alpha_1 t} - \frac{\beta_0}{\beta_2} + B_2 e^{\beta_2 t} + C_2 e^{\gamma_3 t} \\ Y_g &= \bar{Y}_g e^{\gamma_3 t} \end{aligned} \quad \dots \quad (4.32)$$

where

$$\begin{aligned} A_1 &= \left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right\} \\ C_1 &= \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \\ A_2 &= \frac{\alpha_2}{\alpha_1 - \beta_2} \left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right\} \\ B_2 &= \bar{Y}_2 + \frac{\beta_0}{\beta_2} - \frac{\alpha_2}{\alpha_1 - \beta_2} \left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right\} \\ &\quad - \frac{\bar{Y}_g}{\gamma_3 - \beta_2} \left\{ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right\} \\ C_2 &= \frac{\bar{Y}_g}{\gamma_3 - \beta_2} \left\{ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right\} \end{aligned} \quad \dots \quad (4.33)$$

As could be seen from the subsequent numerical analysis,  $C_1$  and  $C_2$  are much smaller in magnitude in relation to  $A_1$  and  $A_2$ . Accordingly as a first approximation we could write  $Y_1$  and  $Y_2$  as

$$Y_1 = A_1 e^{\alpha_1 t}$$

$$Y_2 = A_2 e^{\alpha_1 t} - \frac{\beta_0}{\beta_2} + B_2 e^{\beta_2 t} \quad \dots (4.34)$$

$$\text{Let } y = \frac{Y_2}{Y_1} = \frac{A_2 e^{\alpha_1 t} - \frac{\beta_0}{\beta_2} + B_2 e^{\beta_2 t}}{A_1 e^{\alpha_1 t}}$$

$$\text{i.e. } y = \frac{A_2}{A_1} - \frac{\beta_0}{\beta_2} \frac{e^{-\alpha_1 t}}{A_1} + \frac{B_2}{A_1} e^{(\beta_2 - \alpha_1)t} \quad \dots (4.35)$$

For turning points,

$$\frac{dy}{dt} = \frac{\beta_0 \alpha_1}{A_1 \beta_2} e^{-\alpha_1 t} + \frac{B_2 (\beta_2 - \alpha_1)}{A_1} e^{(\beta_2 - \alpha_1)t} = 0$$

$$\text{i.e. } e^{\beta_2 t} = \frac{\alpha_1 \beta_0}{\beta_2 (\alpha_1 - \beta_2) B_2}$$

$$\text{i.e. } t = \frac{1}{\beta_2} \log_e \left\{ \frac{\alpha_1 \beta_0}{\beta_2 (\alpha_1 - \beta_2) B_2} \right\} \quad \dots (4.36)$$

$$\text{Further, } \frac{d^2 y}{dt^2} = \frac{-\beta_0}{A_1 \beta_2} \alpha_1^2 e^{-\alpha_1 t} + \frac{B_2}{A_1} (\beta_2 - \alpha_1)^2 e^{(\beta_2 - \alpha_1)t}$$

$$\text{When } t = \frac{1}{\beta_2} \log_e \left\{ \frac{\alpha_1 \beta_0 A_1}{\beta_2 (\alpha_1 - \beta_2) B_2} \right\},$$



$$\frac{d^2 y}{dt^2} = e^{-\alpha_1 t} \left\{ -\frac{\beta_0}{A\beta_2} \alpha_1^2 + \frac{B_2}{A_1} (\beta_2 - \alpha_1)^2 \frac{\alpha_1 \beta_0}{\beta_2 (\alpha_1 - \beta_2) B_2} \right\}$$

$$= \frac{-\beta_0 \alpha_1 e^{-\alpha_1 t}}{A_1} > 0 \text{ provided } \beta_0 < 0, (A_1 \alpha_1 > 0).$$

Thus,  $y$  i.e.  $\frac{Y_2}{Y_1}$  will have a minimum point when  $t$  assumes the

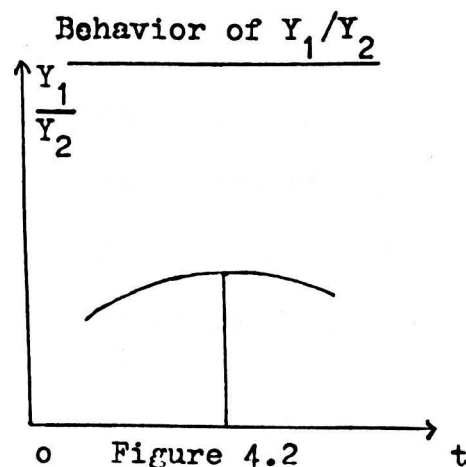
value given by equation (4.36). That is,

$\frac{Y_1}{Y_2}$  will have a maximum point when  $t$

assumes this value. The path traced

by  $\frac{Y_1}{Y_2}$  will thus be of the inverted

"U" shape as shown in Fig. 4.2



Therefore, we may conclude that the model is capable of generating the Kuznets pattern.

The dynamic properties of the model could be best illustrated in numerical terms as in the case of the Chenery-Ahluwalia exercise. They observed that there are no country studies as yet that provide estimates of all parameters required by the model. This is precisely the case in the present exercise - more so in regard to the Sri Lanka situation where the data base suffers from numerous weakness as in the case of many a developing country. Following the Chenery-Ahluwalia approach,

there is no option but to introduce plausible values of parameters. However, in arriving at values of certain parameters, income distribution data and related information were used as best as possible. In instances where difficulties were encountered, the use of plausible values based on judgement was found to be inevitable. One of the specific points we wish to examine is as follows: Assuming that Sri Lanka is a typical developing country, what should be the expected pattern of change of income distribution over time? If this expected pattern has not in fact taken place in the past, we shall then have to explain the departure from the pattern.

Notes and explanations regarding the estimation of the parameters are set out in Appendix V. Let us now consider the first set of values of parameters given below.

Set 1

$$a_1 = 0.30 \quad b_1 = 0.32 \quad b_2 = 0.29 \quad a_g = 0.25$$

$$w_{11} = 0.1 \quad w_{21} = 0.4 \quad p_1 = 0.5$$

$$w_{1g} = 0.2 \quad w_{2g} = 0.6 \quad p_g = 0.2$$

$$s_1 = 0.2 \quad s_0 = -4 \quad s_2 = 0.17 \quad s_g = 0.10$$

$$q = 0.6$$

$$\bar{Y} = 38 \quad \bar{Y}_2 = 52 \quad \bar{Y}_g = 10 \quad (\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_g = 100)$$

i.e.  $\bar{Y}_1, \bar{Y}_2, \bar{Y}_g$  are expressed as percentages of total income Rs.11,300 (see Appendix V for details)

Substitution of these values in equations (4.21) yield

$$\alpha_1 = 0.047 \quad \gamma_1 = 0.005$$

$$\alpha_2 = 0.014 \quad \beta_2 = 0.049 \quad \beta_0 = -1.12$$

$$\gamma_2 = 0.015$$

$$\gamma_3 = 0.005$$

When these values are in turn substituted in equations (4.33) we get

$$A_1 = 39.2 \quad C_1 = -1.2$$

$$A_2 = -274.4 \quad B_2 = 306.9 \quad C_2 = -3.4$$

$$\frac{\beta_0}{\beta_2} = -22.9$$

$$\text{Therefore } Y_g = 10 e^{0.005t}$$

$$Y_1 = 39.2 e^{0.047t} - 1.2 e^{0.005t}$$

$$Y_2 = 22.9 + 306.9 e^{0.049t} - 274.4 e^{0.047t} - 3.4 e^{0.005t}$$

Using the first approximations made in the preceding analysis

$$y = \frac{Y_2}{Y_1} = 0.58 e^{-0.047t} + 7.83 e^{0.002t} - 7.00$$

The condition  $\frac{dy}{dt} = 0$  for a turning point yields

$$e^{0.049t} = 1.738$$

i.e.  $t \approx 11.2$  years

We may check this result with that obtained below.

Giving values  $t = 1, 2, 3$  etc., the values of  $Y_1$  and  $Y_2$  could be calculated and the behaviour of  $\frac{Y_1}{Y_2}$  and  $\frac{Y_1}{Y_1 + Y_2}$  over time could be traced out as follows:

Table 4.2

Changes in values of  $Y_1$  and  $Y_2$  over time (Set 1)

$t$	$Y_1$	$Y_2$	$\frac{Y_1}{Y_2}$	$\frac{Y_1}{Y_1 + Y_2}$
1	39.850	54.191	0.73592	0.4239
2	41.851	56.501	0.74071	0.4255
3	43.917	58.997	0.74439	0.4267
4	46.083	61.629	0.74775	0.4278
5	48.354	64.426	0.75054	0.4287
6	50.734	67.397	0.75276	0.4295
7	53.229	70.553	0.75445	0.4300
8	55.844	73.903	0.75562	0.4304
9	58.585	77.465	0.75628	0.4306
10	61.459	81.245	0.75647	0.4307
11	64.471	85.258	0.75619	0.4306
12	67.628	89.517	0.75548	0.4306
13	70.938	94.037	0.75436	0.4300
14	74.407	98.834	0.75285	0.4295
15	78.043	103.924	0.75096	0.4289
16	81.855	109.324	0.74862	0.4282
17	85.850	115.053	0.74618	0.4273
18	90.037	121.218	0.74332	0.4264
19	94.426	127.570	0.74019	0.4254
20	99.026	134.401	0.73680	0.4242

As seen from Table 4.2, the turning points of  $\frac{Y_1}{Y_2}$  and  $\frac{Y_1}{Y_1 + Y_2}$  are reached at  $t = 10$ , which is close to the result obtained through a first approximation.

In the above table  $(Y_1 + Y_2)$  increases from 90 at  $t = 0$  to 142.704 at  $t = 10$ , representing an increase of 58.56 per cent or an average annual growth rate of approximately 5 per cent. The implication of the result is that if we regard Sri Lanka as a typical developing country with a per capita income of less than US \$200, we should expect income inequality to widen over ten years in reaching a maximum.

Similar results could be obtained by assigning alternative values to the parameters. For instance, if we assign  $b_2$  the value 0.30 instead of 0.28 and leave all other values unchanged,  $\beta_2$  changes to 0.51 and  $\beta_0$  to -1.2, the turning point occurs at  $t = 9$  years approximately. But certain dramatic changes can be observed as the difference between  $\alpha_1$  and  $\beta_2$  widens. This result could be easily deduced from equation (4.36). For, the functional form would indicate that as  $(\beta_2 - \alpha_1)$  increases, a sharp decrease could be expected in  $t$ . In order to illustrate this point let us now consider another set of values of the parameters which differs from the first set only in respect of  $b_2$  and  $s_2$ .

Set 2

$$\bar{Y}_1 = 38 \quad \bar{Y}_2 = 52 \quad \bar{Y}_g = 10$$

$$\begin{aligned} a_1 &= 0.30 & b_1 &= 0.32 & b_2 &= 0.30 & a_g &= 0.25 \\ w_{11} &= 0.10 & w_{21} &= 0.4 & p_1 &= 0.5 \\ w_{1g} &= 0.20 & w_{2g} &= 0.6 & p_2 &= 0.2 \\ s_1 &= 0.20 & s_0 &= -4 & s_2 &= 0.18 & s_g &= 0.10 \\ & & & & & & q &= 0.6 \end{aligned}$$

Substitution of these values in equation (4.21) yields

$$\begin{aligned} \alpha_1 &= 0.047 & \gamma_1 &= 0.005 \\ \alpha_2 &= 0.014 & \beta_2 &= 0.054 & \beta_0 &= -1.2 & \gamma_2 &= 0.015 \\ \gamma_3 &= 0.005 \end{aligned}$$

When these values are in turn substituted in equations (4.33) we get

$$\begin{aligned} A_1 &= 39.2 & C_1 &= -1.2 \\ A_2 &= -78.4 & B_2 &= 111.5 & C_2 &= -3.1 \end{aligned}$$

$$\frac{\beta_0}{\beta_2} = -22$$

$$Y_g = 10 e^{0.005t}$$

$$Y_1 = 39.2 e^{0.047t} - 1.2 e^{0.005t}$$

$$Y_2 = 22.2 + 111.5 e^{0.054t} - 78.4 e^{0.047t} - 3.1 e^{0.005t}$$

Giving values  $t = 1, 2, 3$  etc., the results obtained were as follows:

Table 4.3

Changes in the values of  $Y_1$  and  $Y_2$  over time  
(Set 2)

t	$Y_1$	$Y_2$	$\frac{Y_1}{Y_2}$	$\frac{Y_1}{Y_1 + Y_2}$
1	39.880	54.398	0.73312	0.4230
2	41.851	56.958	0.73477	0.4236
3	43.917	59.689	0.73576	0.4236
4	46.083	62.604	0.73610	0.4240
5	48.354	65.713	0.73584	0.4239
6	50.734	69.029	0.73497	0.4236
7	53.229	72.565	0.73354	0.4231
8	55.844	76.335	0.73156	0.4225
9	58.585	80.353	0.72910	0.4217
10	61.459	84.636	0.72616	0.4207

As seen from the above table, the turning point occurs at  $t = 4$ . Similarly, if the exercise is repeated with  $s_1$  set at 0.19 instead of 0.20, the turning point occurs at almost  $t = 0$ . In other words, changes that increase  $(\beta_2 - \alpha_1)$  tend to shorten the time taken to reach the turning point; if the changes are large enough, the time taken may be insignificant, or may even take a negative value, indicating that the turning point has already taken place. In such a situation the Kuznets pattern

(the inverted "U" shape of income inequality over time) could become indiscernible and the dominant pattern produced by the model would be one of a steady trend towards equality for the period under consideration. The indiscernability of the Kuznets pattern is also an extreme case that the model ought to contain. However, this special feature could be traceable to strong redistributive forces. Accordingly we shall in the next section, generalise the model a step further so as to accommodate strong and explicit redistributive measures. The object of this generalisation is to develop the model so that whilst it will be capable of producing the Kuznets pattern in general it will also be possible to generate an extreme case that could correspond to the Sri Lanka context. For, income inequality in Sri Lanka has diminished in the recent past although in terms of the Kuznets pattern, income inequality should have increased since the per capita income in Sri Lanka was well below the US \$200 level in the 1960s, on the assumption that the estimates of the relevant parameters are approximately correct.

#### 4.8 Growth with further Redistributive Measures.

As noted in section 4.3 the model has been developed on the assumption that certain redistributive measures were operative. In this section we shall introduce explicit redistributive measures into the model. In order to do this we shall have to



go back to equations (4.7) and effect the following modifications:

$$\begin{aligned}
 Y_1 &= [(w_{11} + p_1) a_1 K_1^l + b_1 K_1^n + w_{1g} a_g K_g^l](1 - \lambda) \\
 Y_2 &= w_{21} a_1 K_1^l + b_2 K_1^n + w_{2g} a_g K_g^l + \\
 &\quad \lambda [(w_{11} + p_1) a_1 K_1^l + b_1 K_1^n + w_{1g} a_g K_g^l] \\
 Y_g &= p_g a_g K_g^l \quad \dots \quad (4.37)
 \end{aligned}$$

The first two equations represent a transfer of a proportion  $\lambda$  of the income of the rich to the poor; government income  $Y_g$  remains unaffected. Thus  $Y_1$ ,  $Y_2$ ,  $Y_g$  now represent incomes resulting from this transformation. An implicit assumption here is that the rich and the poor do not reverse roles over the time span considered. The plausibility of this assumption is shown by the results of section 4.8 and of Chapter 7.

The capital accumulation equations are:

$$\begin{aligned}
 \dot{K}_1^l &= q s_1 Y_1 \\
 \dot{K}_1^n &= (1 - q) s_1 Y_1 \\
 \dot{K}_2^n &= s_0 + s_2 Y_2 \\
 \dot{K}_g^l &= s_g Y_g \quad \dots \quad (4.38)
 \end{aligned}$$

Differentiating equations (4.37) with respect to  $t$ , eliminating the  $\dot{K}$  s by the use of equations (4.38) and by further simplification, we get:

$$\begin{aligned}
\dot{Y}_1 &= \alpha_1 (1 - \lambda) Y_1 + \gamma_1 (1 - \lambda) Y_g \\
\dot{Y}_2 &= (\alpha_2 + \lambda \alpha_1) Y_1 + \beta_0 + \beta_2 Y_2 + (\gamma_2 + \lambda \gamma_1) Y_g \\
\dot{Y}_g &= \gamma_3 Y_g \quad \dots \quad (4.39)
\end{aligned}$$

where  $\alpha_1, \gamma_1, \alpha_2, \beta_0, \beta_2, \gamma_2, \gamma_3$  are defined as before by equations (4.21)

Equations (4.39) are clearly of the same form as equations (4.20); the only change relates to the parameters. If we denote the new parameters by  $\alpha'_1, \gamma'_1, \alpha'_2$  and so on, then the changes in the parameters could be represented in matrix form as follows:

$$\begin{bmatrix} \alpha'_1 \\ \gamma'_1 \\ \alpha'_2 \\ \beta'_0 \\ \beta'_2 \\ \gamma'_2 \\ \gamma'_3 \end{bmatrix} = \begin{bmatrix} (1 - \lambda) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1 - \lambda) & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \gamma_1 \\ \alpha_2 \\ \beta_0 \\ \beta_2 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \quad \dots \quad (4.40)$$

The presence of only two non-diagonal elements in the above matrix indicates that the income transfer considered here is about the simplest possible. It is also clear that when we set  $\lambda = 0$ , we get back the previous model as a particular

case. In this sense, the above model could be regarded as a generalization of the previous model by one step.

The behaviour pattern of income inequality in Sri Lanka in the recent past, namely the steady trend towards equality could be obtained from the model by hypothesising as follows: Redistributive policies in Sri Lanka have been sufficiently strong so that  $\lambda$  which represents their impact assumes a high value. As a result,  $\alpha_1'$  diminishes in relation to  $\alpha_1$  by a factor  $(1 - \lambda)$ . Therefore the factor  $(\beta_2 - \alpha_1')$  increases in relation to  $(\beta_2 - \alpha_1)$ . The net effect of this change is to advance the time taken for the turning point so that the Kuznets pattern becomes indiscernible during the period concerned. Thus the dominant feature is one of a steady trend towards equality.

In order to test this argument in a proper manner, we should in the first instance obtain a satisfactory estimate of  $\lambda$ . Such an estimate has to take account of several redistributive policies such as taxes, pricing of agricultural products, wage policies and other social welfare measures. In view of the obvious difficulties associated with such an exercise, we shall in the alternative assign a few plausible values to  $\lambda$  and study the dynamic properties of the model. These values of will be considered along with the values of parameters in

Set 2 of the previous section.

Case I       $\lambda = 0.1$

$$\alpha'_1 = \alpha_1(1 - \lambda) = 0.047 \times 0.9 = 0.042$$

$$\gamma'_1 = \gamma_1(1 - \lambda) = 0.005 \times 0.9 = 0.0045$$

$$\alpha'_2 = \alpha_2 + \lambda\alpha_1 = 0.014 + 0.047 \times 0.1 = 0.019$$

$$\gamma'_2 = \gamma_2 + \lambda\gamma_1 = 0.015 + 0.005 \times 0.1 = 0.016$$

The transformation (4.40) leaves the other parameters unchanged.

Accordingly,

$$\beta'_2 = 0.054, \quad \beta'_0 = -1.2, \quad \gamma'_3 = 0.005$$

On the basis of the 1963 income distribution pattern (see Table 2.2 in Chapter 2), the following estimates of  $\bar{Y}_1$ ,  $\bar{Y}_2$ ,  $\bar{Y}_g$  seem plausible:

$$\bar{Y}_1 = 47, \quad \bar{Y}_2 = 43, \quad \bar{Y}_g = 10$$

Using equations (4.33), we get

$$B_2 = 100$$

From equation (4.36), the time taken to reach the turning point is given by

$$\begin{aligned} t &= \frac{1}{\beta'_2} \log_e \frac{\alpha'_1 \beta'_0}{\beta'_2 (\alpha'_1 - \beta'_2)} \\ &= \frac{1}{0.054} \log_e (0.778) \\ &= \frac{1}{0.054} (-0.25) = -4.6 \text{ years} \quad \dots (4.41) \end{aligned}$$

The interpretation of this result is that the turning point has already occurred. Therefore the pattern exhibited from  $t = 0$  onwards will be one of a steady trend towards equality.

Case II  $\lambda = 0.15$

$$\text{Then } \alpha'_1 = \alpha_1 (1 - \lambda) = 0.047 \times 0.85 = 0.04$$

$$\gamma'_1 = \gamma_1 (1 - \lambda) = 0.005 \times 0.85 = 0.0043$$

$$\alpha'_2 = \alpha_2 + \lambda \alpha_1 = 0.014 + 0.047 \times 0.15 = 0.021$$

$$\gamma'_2 = \gamma_2 + \lambda \gamma_1 = 0.015 + 0.005 \times 0.15 = 0.016$$

$$\text{As before, } \beta'_2 = 0.054, \quad \beta'_0 = -1.2, \quad \gamma'_3 = 0.005$$

$$\text{and } \bar{Y}_1 = 47, \quad \bar{Y}_2 = 43, \quad \bar{Y}_g = 10$$

Using equations (4.33) we get

$$B_2 = 96$$

From equation (4.36), the time taken to reach the turning point is given by

$$\begin{aligned} t &= \frac{1}{\beta'_2} \log_e \left\{ \frac{\alpha'_1 \beta'_0}{\beta_2 (\alpha'_1 - \beta'_2) B_2} \right\} \\ &= \frac{1}{0.054} \log_e (0.662) \\ &= \frac{1}{0.054} (-0.412) \\ &= -7.6 \text{ years} \end{aligned} \quad \dots (4.42)$$

Once again the result indicates that the turning point has already occurred. As in Case I, the pattern exhibited from  $t = 0$  onwards is one of a steady trend towards equality.

Similar results could be obtained by assigning other values to  $\lambda$ .

It was shown in the previous section that the model is capable of predicting the empirically observed Kuznets pattern. If Sri Lanka was a typical developing country, in terms of the Kuznets pattern, income inequality should have increased in the recent past since the per capita income level was well below the US \$ 200 level assuming that our estimates of the relevant parameters are approximately correct. However, the actual pattern observed in the case of Sri Lanka is a steady trend towards reduction of income inequality in the recent past. This pattern has been achieved through the implementation of a number of redistributive measures in the recent past. On generalising the model in this section so as to include specific redistributive measures, it is found that the model is capable of generating the Sri Lanka pattern too. This takes place through an advancing of the turning point so that the behaviour predicted by the model for the period in question is one of decreasing inequality.

Let us now proceed to investigate a more important aspect, namely the behaviour of total income as defined by  $Y_1 + Y_2$  with respect to increasing values of  $\lambda$ . This could be carried out by assigning different values to  $\lambda$  in equation (4.39) and obtaining the respec-

tive solutions. But these equations are of the same form as equation (4.20); therefore we could use the solutions of equations (4.20) as defined by equations (4.32) and (4.33) with parameters appropriately transformed by equation (4.40).

The general form of the basic solution is:

$$Y_1 = A_1 e^{\alpha_1 t} + C_1 e^{\gamma_1 t}$$

$$Y_2 = A_2 e^{\alpha_2 t} - \frac{\beta_0}{\beta_2} + B_2 e^{\beta_2 t} + C_2 e^{\gamma_2 t}$$

$$Y_g = \bar{Y}_g e^{\gamma_3 t} \quad \dots \quad (4.43)$$

Where  $A_1$ ,  $C_1$ ,  $A_2$ ,  $B_2$  and  $C_2$  are defined by equations (4.33) in which the parameters  $\alpha_1$ ,  $\gamma_1$ ,  $\alpha_2$ ,  $\beta_0$ ,  $\beta_2$ ,  $\gamma_2$  and  $\gamma_3$  assume values in terms of transformation (4.40). As seen from Case I and II discussed above this transformation changes the values of  $\alpha_1$ ,  $\gamma_1$ ,  $\alpha_2$  and  $\gamma_2$  but leaves the values of  $\beta_0$ ,  $\beta_2$ , and  $\gamma_3$  unchanged.

By assigning  $\lambda$  the values of 0.05, 0.10, 0.15, 0.20 and 0.25 in succession, the coefficients of equation (4.43) were calculated and the results are shown in Table 4.4.

Table 4.4Coefficients of Equation (4.43)

$\lambda$	$A_1$	$C_1$	$A_2$	$B_2$	$C_2$	$\alpha_1$	$\beta_2$	$\gamma_3$
0.05	39.192	-1.192	39.006	-21.399	-5.607	0.04484	0.028	0.005
0.10	39.201	-1.201	51.763	-34.350	-5.413	0.04248	0.028	0.005
0.15	39.210	-1.210	69.491	-52.299	-5.192	0.04012	0.028	0.005
0.20	39.221	-1.221	95.802	-78.862	-4.940	0.03776	0.028	0.005
0.25	39.234	-1.234	138.910	-122.262	-4.648	0.03540	0.028	0.005

(Note: The values of the basic parameters used are the same as those of Set 1, except for  $b_2 = 0.28$  and  $s_2 = 0.1$ . The change in value of the latter parameters will enable us to see more clearly the consequences of appreciable differences between the savings rates of the rich and the poor).

Substituting the above values of the coefficients in equation (4.43), and setting  $t = 0$ , the respective values of  $Y_1$ ,  $Y_2$  ( $Y_1 + Y_2$ ) and  $Y_1/Y_2$  were obtained. The results are shown in Table 4.5 and the corresponding graphs in Figures 4.3, 4.4 and 4.5. As evident from equation (4.43) there is no change in  $Y_g$  with respect to  $\lambda$ . It should be noted that the results in Table 4.5 are not strictly comparable with those in Tables 4.2 and 4.3 because of the difference in the parameter  $s_2$ . In section 4.6 the values of  $s_2$  were taken as 0.17 and 0.18 respectively, whereas in the present exercise the value was taken as 0.1 in order to demonstrate more clearly the consequences of a wide difference between the savings rates of the rich and the poor.



Table 4.5

Values of  $Y_1$ ,  $Y_2$ ,  $(Y_1 + Y_2)$ ,  $(Y_1/Y_2)$  at  $t = 10$

$\lambda$	$Y_1$	$Y_2$	$Y_1 + Y_2$	$Y_1/Y_2$
0.05	60.090	66.843	126.933	0.899
0.10	58.686	68.019	126.705	0.863
0.15	57.293	69.137	126.430	0.829
0.20	55.931	70.216	126.147	0.797
0.25	54.602	71.257	125.859	0.766

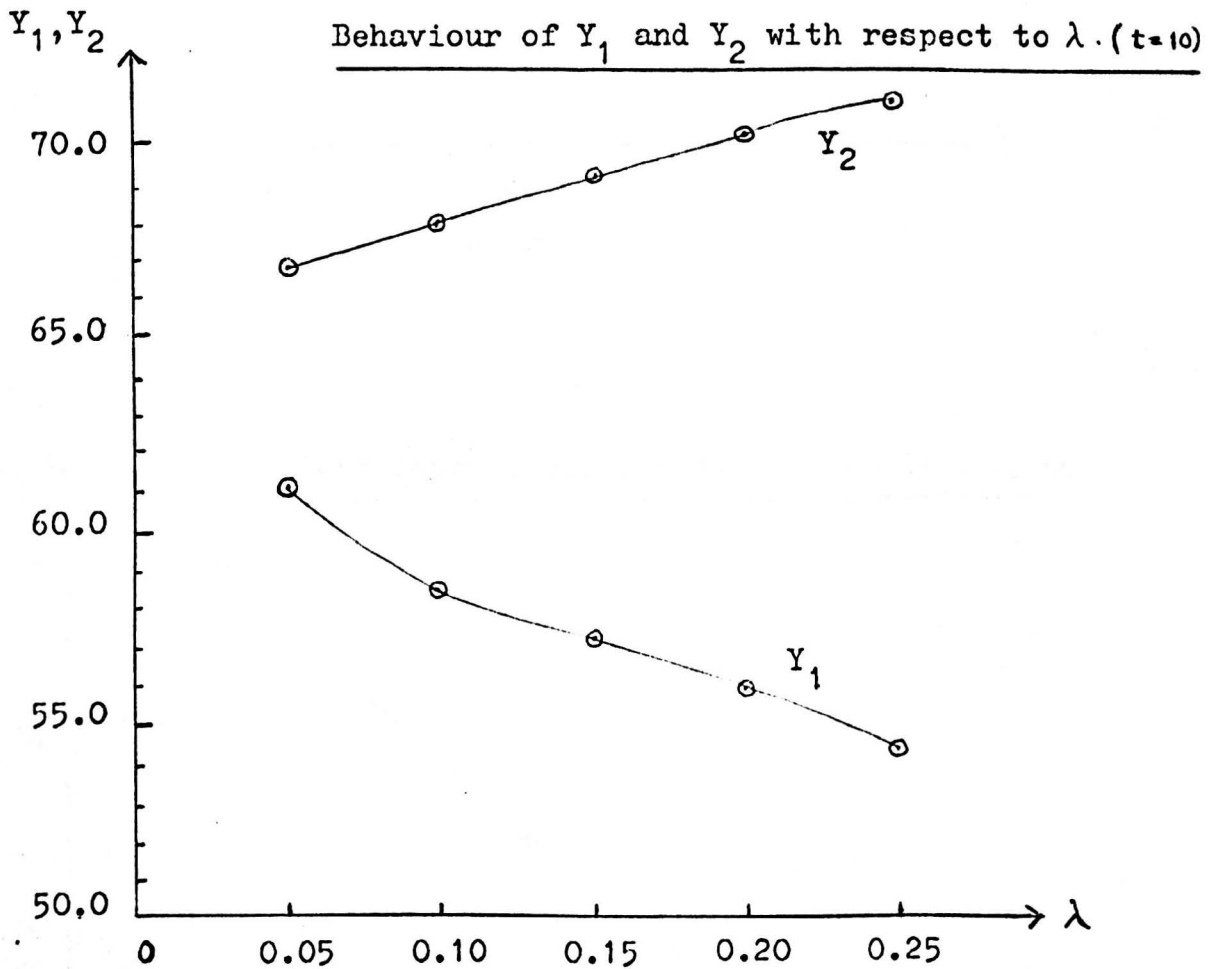


Figure 4.3

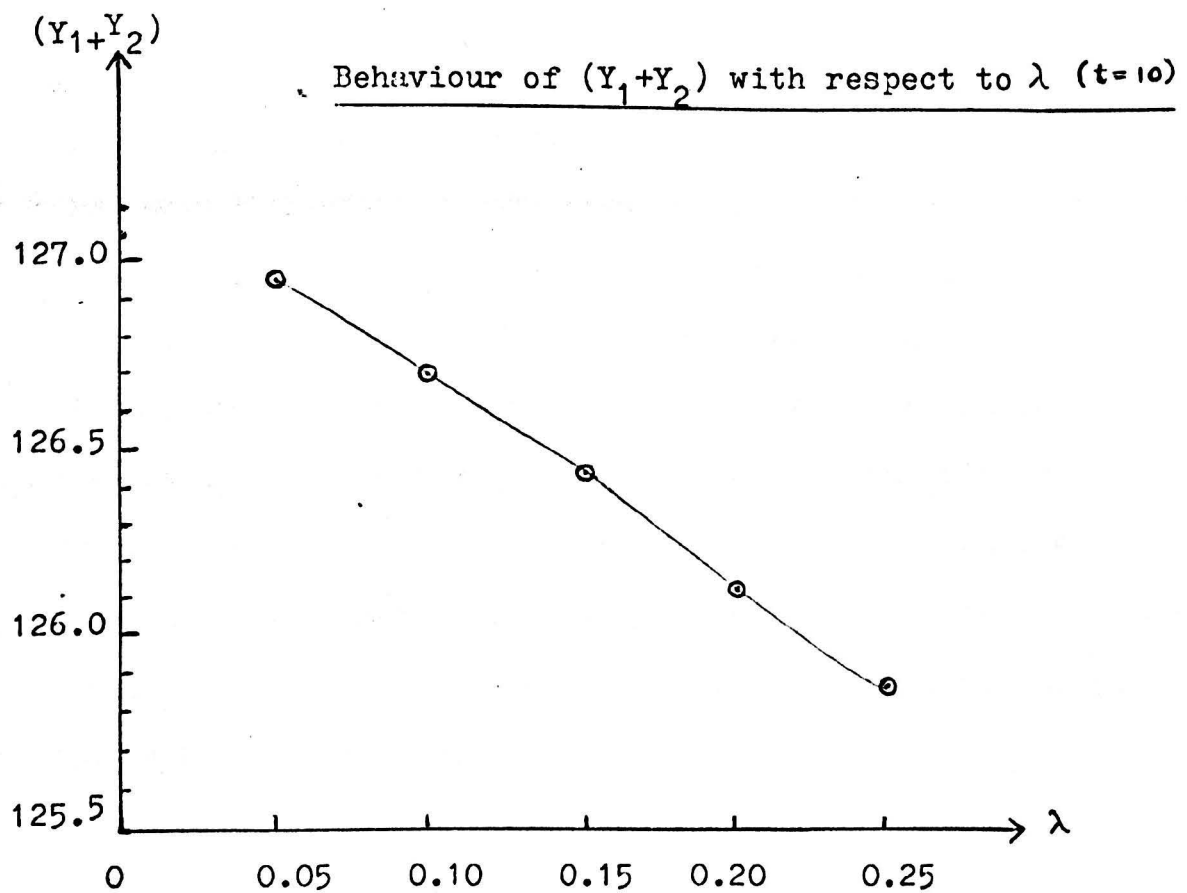


Figure 4.4

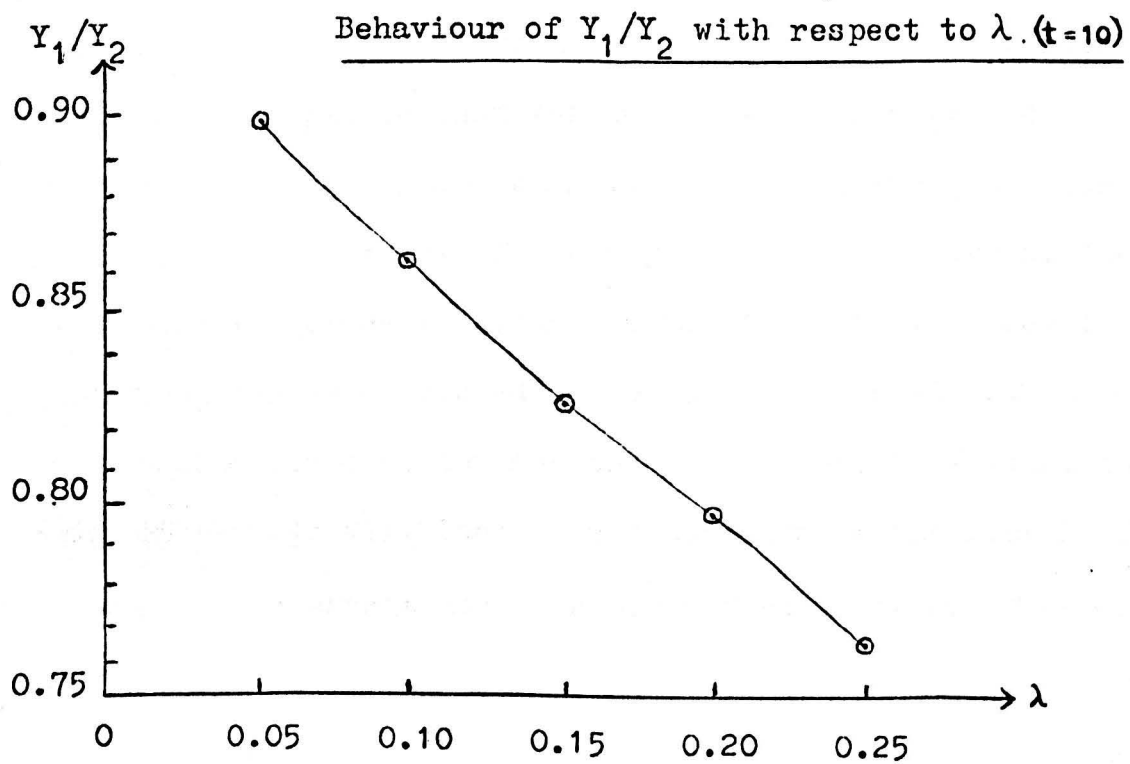


Figure 4.5

As shown in Figure 4.3, the behaviour of  $Y_1$  with increasing  $\lambda$  is one of a steady decline; in contrast  $Y_2$  steadily increases. Figure 4.5 shows that the level of income inequality as represented by  $Y_1/Y_2$  declines steadily as  $\lambda$  increases. Total income too, as represented by  $(Y_1 + Y_2)$  declines steadily with  $\lambda$ . The overall relationship we obtain is one of a decline in total income with increasing income redistribution from the rich to the poor. Thus, income redistribution and growth of total income emerge as competing objectives. The underlying reason is the diminution in total savings due to the difference in the rates of savings of the rich and the poor.

#### 4.9 Redistribution of Consumption

The system of redistribution considered in the previous section essentially consists of a direct transfer of income from the rich to the poor. One immediate consequence of such a transfer is that there would be a loss in total savings of the community. For, we have assumed that the marginal savings rate of the rich is higher than the marginal savings rate of the poor (i.e.  $s_1 > s_2$ ). As a result, for every sum  $\Delta Y_1$  transferred, the loss in total savings would amount to  $\Delta Y_1 (s_1 - s_2)$ . Since the growth processes discussed in terms of our model have been essentially based on savings and investment, it follows that a loss of savings will lead to a diminution in the growth of total income. This effect could be quantified in terms of our model



	<u>Income</u>	<u>Consumption</u>	<u>Savings</u>
The rich	$-\Delta Y_1$	$-c_1 \Delta Y_1$	$-(1 - c_1)\Delta Y_1$
The poor	$\frac{c_1}{c_2} \Delta Y_1$	$c_1 \Delta Y_1$	$(1 - c_2)\frac{c_1}{c_2} \Delta Y_1$
The Government	$(1 - \frac{c_1}{c_2}) \Delta Y_1$	0	$(1 - \frac{c_1}{c_2}) \Delta Y_1$

An implicit assumption here is that there already exists a system of direct and indirect taxes and other sources of income to meet the recurrent expenditure of the government and that any income in excess of this could be completely saved and invested. This assumption could be somewhat relaxed as shown by Codippily {1974} and the general lines of the argument will be essentially the same if in place of the government we introduce a corporate sector with a high marginal propensity to save.

It could also be shown (see Codippily {1974 ; pp. 22-23} ) that the above system of redistribution leads to an improvement in total welfare<sup>based on consumption</sup> as long as the marginal utility of consumption of the poor exceeds that of the rich. The latter condition would in fact hold good if the utility function is concave as generally assumed in the literature. Since a system of consumption redistribution could lead to a welfare improvement and at the same time leave total savings unchanged, we shall, as a next step attempt to incorporate this system into our model.

Before we proceed further, there is one point that needs to be explained. Although a scheme of consumption redistribution ensures no losses in total savings, it does not follow that the growth rate of total income will also remain unchanged. For, the output-capital ratios relating to investments made by the rich, the poor and the government differ in general. Therefore, in general there would be changes in the growth of total income although total savings remain unchanged.

In order to incorporate a system of consumption redistribution into our model we have to go back to equations (4.37) and modify these equations as follows:

$$\begin{aligned}
 Y_1 &= [(w_{11} + p_1) a_1 K_1^l + b_1 K_1^n + w_{1g} a_g K_g^l] (1 - \lambda) \\
 Y_2 &= w_{21} a_1 K_1^l + b_2 K_2^n + w_{2g} a_g K_g^l \\
 &\quad + \frac{c_1 \lambda}{c_2} [(w_{11} + p_1) a_1 K_1^l + b_1 K_1^n + w_{1g} a_g K_g^l] \\
 Y_g &= p_g a_g K_g^l + (1 - \frac{c_1}{c_2}) \lambda [(w_{11} + p_1) a_1 K_1^l + b_1 K_1^n \\
 &\quad + w_{1g} a_g K_g^l] \quad \dots \quad (4.44)
 \end{aligned}$$

In view of the special condition that the amount retained by government goes directly into investment, the capital accumulation equations have to be modified as follows:

$$\begin{aligned}
\dot{K}_1^l &= q s_1 Y_1 \\
\dot{K}_1 &= (1 - q) s_1 Y_1 \\
\dot{K}_2^n &= s_0 + s_2 Y_2 \\
\dot{K}_g^l &= s_g (p_g a_g K_g^l) + (1 - \frac{c_1}{c_2}) \lambda [(w_{11} + p_1) a_1 K_1^l \\
&\quad + b_1 K_1^n + w_{1g} a_g K_g^l] \quad \dots (4.45)
\end{aligned}$$

Using the last of equations (4.44), the factor  $p_g a_g K_g$  could be eliminated and the last of equations (4.45) could be written as

$$\begin{aligned}
\dot{K}_g^l &= s_g \left[ Y_g - (1 - \frac{c_1}{c_2}) \lambda \left\{ (w_{11} + p_1) a_1 K_1^l + b_1 K_1^n \right. \right. \\
&\quad \left. \left. + w_{1g} a_g K_g^l \right\} \right] \\
&\quad + (1 - \frac{c_1}{c_2}) \lambda \left[ (w_{11} + p_1) a_1 K_1^l + b_1 K_1^n \right. \\
&\quad \left. + w_{1g} a_g K_g^l \right] \quad \dots (4.46)
\end{aligned}$$

But from the first of equations (4.44), it is clear that

$$(w_{11} + p_1) a_1 K_1^l + b_1 K_1^n + w_{1g} a_g K_g^l = \frac{1}{1 - \lambda} Y_1 \quad \dots (4.47)$$

Therefore equation (4.46) could be written as

$$\begin{aligned}
\dot{K}_g^l &= s_g \left[ Y_g - (1 - \frac{c_1}{c_2}) \frac{\lambda}{1 - \lambda} Y_1 \right] + (1 - \frac{c_1}{c_2}) \frac{\lambda}{1 - \lambda} Y_1 \\
\text{i.e. } \dot{K}_g^l &= s_g Y_g + (1 - s_g) (1 - \frac{c_1}{c_2}) \frac{\lambda}{1 - \lambda} Y_1 \quad \dots (4.48)
\end{aligned}$$

Although  $c_1$  and  $c_2$  differed in the equations of the previous sections as well, yet this difference was not reflected in

equation (4.37). Therefore the particular case of  $c_1 = c_2$  would leave equation (4.37) unaffected. If we set  $c_1 = c_2$  in the above equations, it could be seen that we get back equations (4.37) and (4.38). In this sense, the above equations could be regarded as a further generalization of the equations (4.37) and (4.38).

Differentiating equations (4.44), eliminating the  $\dot{K}$  s by use of equations (4.45) and (4.48) and by further simplification we get

$$\begin{aligned}\dot{Y}_1 &= [\alpha_1 (1 - \lambda) + \eta \lambda \gamma_1] Y_1 + (1 - \lambda) \gamma_1 Y_g \\ \dot{Y}_2 &= [\alpha_2 + \frac{c_1 \lambda \alpha_1}{c_2} + \frac{\eta \lambda}{1 - \lambda} \gamma_2 + \frac{\eta c_1 \lambda^2}{c_2 (1 - \lambda)} \gamma_1] Y_1 + \beta_0 + \beta_2 Y_2 \\ &\quad + [\gamma_2 + \frac{c_1 \lambda \gamma_1}{c_2}] Y_g \\ \dot{Y}_g &= [\gamma_3 + (1 - \frac{c_1}{c_2}) \lambda \gamma_1] Y_g + [(1 - \frac{c_1}{c_2}) \lambda \alpha_1 + \frac{\eta \lambda}{1 - \lambda} \gamma_3 \\ &\quad + \frac{\eta \lambda^2}{1 - \lambda} (1 - \frac{c_1}{c_2}) \gamma_1] Y_1 \dots (4.49)\end{aligned}$$

where  $\alpha_1, \gamma_1, \alpha_2, \beta_0, \beta_2, \gamma_2, \gamma_3$  are defined as before by equation (4.21) and

$$\eta = \left\{ \frac{1}{s_g} - 1 \right\} (1 - \frac{c_1}{c_2}) \dots (4.50)$$

Again, if set  $c_1 = c_2$ , then  $\eta = 0$ , and consequently equations (4.49) reduce to equation (4.39)



Basically, the above equations are of the same form as equations (4.20), but as in the case of equation (4.39), the parameters have changed. If we denote the new parameters by  $\alpha'_1$ ,  $\gamma'_1$ ,  $\alpha'_2$  and so on, then the changes in the parameters could be represented in matrix form as follows:

$$\begin{bmatrix} \alpha'_1 \\ \gamma'_1 \\ \alpha'_2 \\ \beta'_0 \\ \beta'_2 \\ \gamma'_2 \\ \alpha'_3 \\ \gamma'_3 \end{bmatrix} = \begin{bmatrix} 1-\lambda & \eta\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{c_1}{c_2} \lambda \alpha_1 & \frac{c_1 \eta \lambda^2}{c_2 (1-\lambda)} & 1 & 0 & 0 & \frac{\eta \lambda}{1-\lambda} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{c_1 \lambda}{c_2} & 0 & 0 & 0 & 1 & 0 & 0 \\ (1-\frac{c_1}{c_2})\lambda & \frac{\eta \lambda^2}{1-\lambda} (1-\frac{c_1}{c_2}) & 0 & 0 & 0 & 0 & 1 & \frac{\eta \lambda}{1-\lambda} \\ 0 & (1-\frac{c_1}{c_2})\lambda & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \gamma_1 \\ \alpha_2 \\ \beta_0 \\ \beta_2 \\ \gamma_2 \\ 0 \\ \gamma_3 \end{bmatrix} \quad \dots(4.51)$$

The matrix in the above transformation is clearly more complex than that of equation (4.40); the number of non diagonal elements have increased from 2 to 9 and a new parameter  $\gamma_3$  has emerged. The latter presents a further difficulty as would be

seen shortly in the solution of set of simultaneous differential equations, which were hitherto somewhat straightforward. The above matrix reflects that redistribution of consumption is relatively more complex than redistribution of income and also indicates that the results could be appreciably different. As before, we could assign numerical values to the parameters and investigate the nature of the solutions of the respective differential equations. Equations (4.49) can be written as follows in terms of the new parameters defined by the transformation (4.51).

$$\begin{aligned}\dot{Y}_1 &= \alpha'_1 Y_1 + \gamma'_1 Y_g \\ \dot{Y}_2 &= \alpha'_2 Y_1 + \beta'_0 + \beta'_2 Y_2 + \gamma'_2 Y_g \\ \dot{Y}_g &= \alpha'_3 Y_1 + \gamma'_3 Y_g \quad \dots \quad (4.52)\end{aligned}$$

In the case of equations (4.20) and (4.39) the method of solution was somewhat simpler since there was no term in  $Y_1$  in the last equation. This made it possible to solve the last equation to begin with, substitute the solution of  $Y_g$  in the first and obtain a solution  $Y_1$ . Thereafter, the solution of  $Y_1$  and  $Y_g$  were substituted in the second equation and the solution for  $Y_2$  was obtained. But in the above equations the presence of the term in  $Y_1$  in the last equation precludes us from using such a procedure. In these circumstances the first step would be solve the first and the last simultaneously. Since the problem

is essentially an initial value problem, a convenient approach is to use the Laplace transform as defined by

$$\mathcal{L}\{F(t)\} = \int_0^{\infty} e^{-st} F(t) dt = f(s) \quad \dots (4.53)$$

Two of its well known properties which will be of use in solving equations of the type (4.52) are:

$$\mathcal{L}\{\dot{F}\} = sf(s) - F(0)$$

$$\text{and } \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad \dots (4.54)$$

$$\text{Let } \mathcal{L}(y_1) = \bar{y}_1 \quad \text{and} \quad \mathcal{L}(y_g) = \bar{y}_g$$

Applying the Laplace transform to the first and the last of equations (4.52) we get

$$s\bar{y}_1 - \bar{y}_1 = \alpha'_1 \bar{y}_1 + \gamma'_1 \bar{y}_g$$

$$\text{and } s\bar{y}_g - \bar{y}_g = \alpha'_3 \bar{y}_1 + \gamma'_3 \bar{y}_g \quad \dots (4.55)$$

Solving for  $y_1$  and  $y_2$  we get

$$\bar{y}_1 = \frac{\bar{y}_1 (s - \gamma'_3) + \bar{y}_g \gamma'_1}{(s - \alpha'_1)(s - \gamma'_3) - \gamma'_1 \alpha'_3}$$

$$\bar{y}_g = \frac{\bar{y}_1 \gamma'_3 + \bar{y}_g (s - \alpha'_1)}{(s - \alpha'_1)(s - \gamma'_3) - \gamma'_1 \alpha'_3} \quad \dots (4.56)$$

When numerical values are assigned to  $\alpha'_1$ ,  $\gamma'_1$ ,  $\alpha'_3$  and  $\gamma'_3$ ,

the above equations can be put in the partial fraction form

$$y = \frac{A'}{(s - a')} + \frac{B'}{(s - b')} \quad \dots (4.57)$$

where  $A'$  and  $B'$  are constant and  $(s - a')$  and  $(s - b')$  are the factors of the quadratic  $\{(s - \alpha'_1)(s - \alpha'_3) - \gamma'_1 \alpha'_3\}$ . Using the second property defined by equations (4.54), the solutions of  $Y_1$  and  $Y_g$  can be obtained directly. Substituting there solutions in the second of equations (4.52), the solution of  $Y_2$  could be obtained. Appendix (VI) presents the numerical solution corresponding to each of the cases  $\lambda = 0.05, 0.10, \dots 0.25$ . The results obtained in each case are set out below:

Case I       $\lambda = 0.05$

$$\begin{aligned} Y_1 &= 39.125e^{0.04516t} - 1.125e^{0.00497t} \\ Y_2 &= 40 + 39.866e^{0.04516t} - 22.430e^{0.028t} \\ &\quad - 5.436e^{0.00497t} \\ Y_g &= 0.515e^{0.04516t} + 9.485e^{0.00497t} \quad \dots (4.58) \end{aligned}$$

Case II       $\lambda = 0.10$

$$\begin{aligned} Y_1 &= 39.059e^{0.04311t} - 1.059e^{0.00493t} \\ Y_2 &= 40 + 53.600e^{0.04311t} - 36.576e^{0.028t} \\ &\quad - 5.024e^{0.00493t} \\ Y_g &= 1.104e^{0.04311t} + 8.896e^{0.00493t} \quad \dots (5.59) \end{aligned}$$

Case III  $\lambda = 0.15$ 

$$\begin{aligned}
 Y_1 &= 38.964e^{0.04107t} - 0.964e^{0.00488t} \\
 Y_2 &= 40 + 72.242e^{0.04107t} - 55.679e^{0.028t} \\
 &\quad - 4.563e^{0.00488t} \dots (4.60) \\
 Y_g &= 1.822e^{0.04107t} + 8.178e^{0.00488t} \dots (4.60)
 \end{aligned}$$

Case IV  $\lambda = 0.20$ 

$$\begin{aligned}
 Y_1 &= 38.881e^{0.03903t} - 0.881e^{0.00485t} \\
 Y_2 &= 40 + 98.277e^{0.03903t} - 82.262e^{0.028t} \\
 &\quad - 4.015e^{0.00485t} \\
 Y_g &= 2.665e^{0.03903t} + 7.335e^{0.00485t} \dots (4.61)
 \end{aligned}$$

Case V  $\lambda = 0.25$ 

$$\begin{aligned}
 Y_1 &= 38.751e^{0.03701t} - 0.751e^{0.00479t} \\
 Y_2 &= 40 + 136.848e^{0.03701t} - 121.442e^{0.028t} \\
 &\quad - 3.406e^{0.00479t} \\
 Y_g &= 3.681e^{0.03701t} + 6.319e^{0.00479t} \dots (4.62)
 \end{aligned}$$

Setting  $t = 10$  in the above equations the respective values of  $Y_1$ ,  $Y_2$ ,  $(Y_1 + Y_2)$  and  $Y_1/Y_2$  were obtained. The results are shown in Table 4.6 and the corresponding graphs in Figures 4.6 4.7 and 4.8.

Table 4.6

Values of  $Y_1$ ,  $Y_2$ ,  $(Y_1 + Y_2)$ ,  $(Y_1/Y_2)$  at  $t = 10$

$\lambda$	$Y_1$	$Y_2$	$Y_1 + Y_2$	$Y_1/Y_2$
0.05	60.276	67.194	127.470	0.897
0.10	58.997	68.815	127.812	0.857
0.15	57.741	70.470	128.211	0.819
0.20	56.519	72.138	128.657	0.783
0.25	55.319	73.883	129.202	0.749

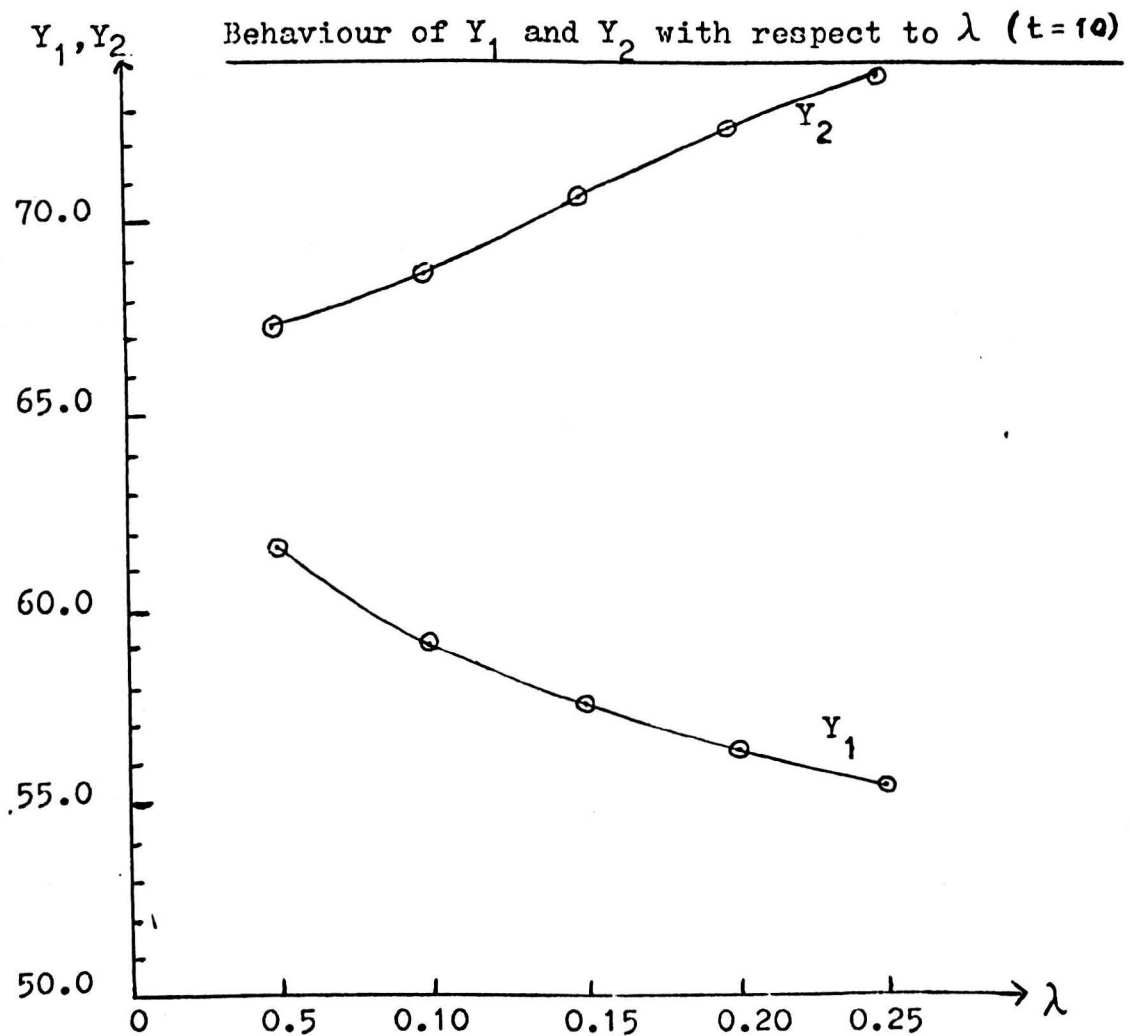


Figure 4.6

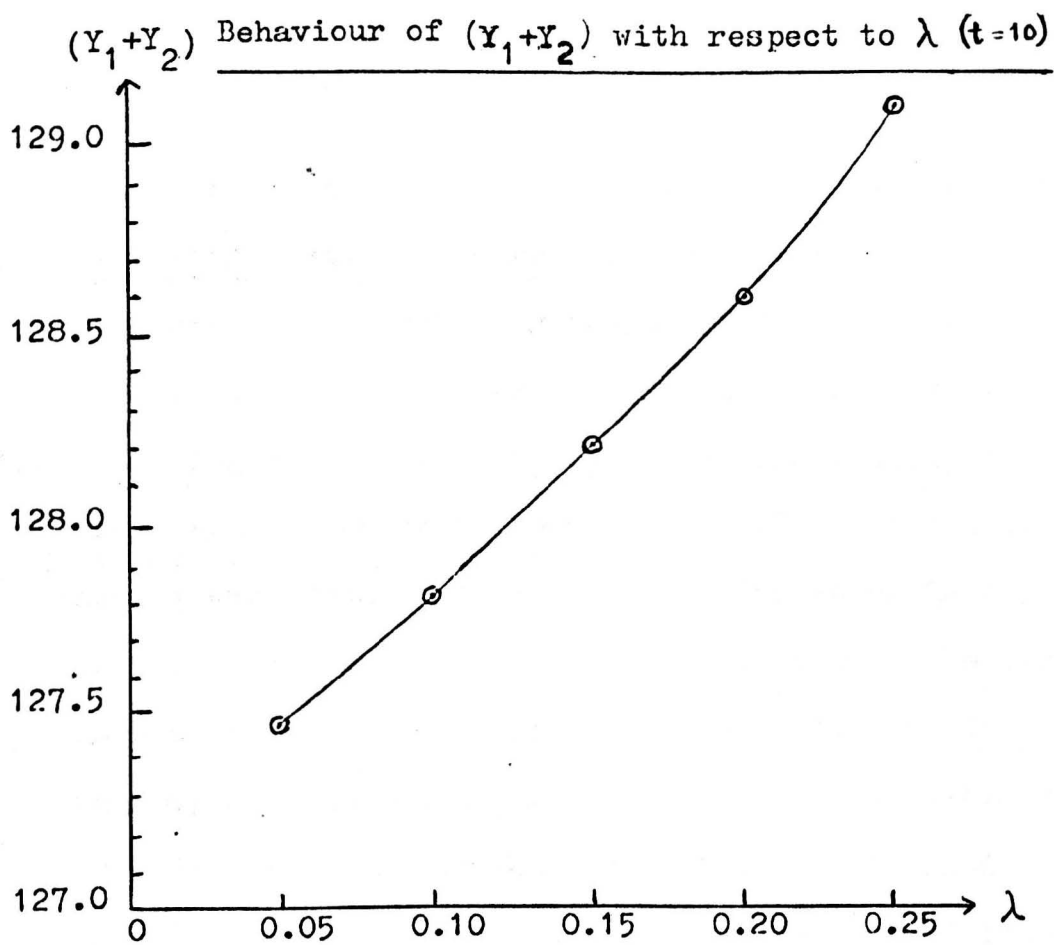


Figure 4.7

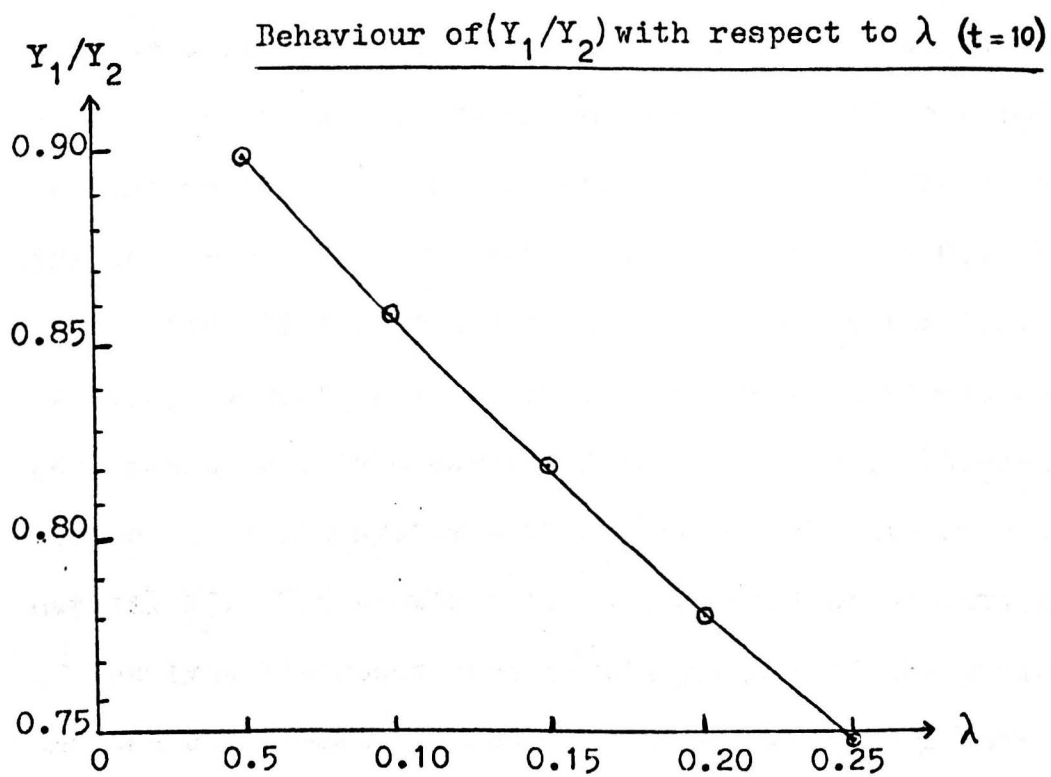


Figure 4.8

As in the case of income redistribution discussed in the preceding section, Figure 4.6 shows that  $Y_1$  declines while  $Y_2$  increases with increasing values of  $\lambda$ , in the case of consumption redistribution. But a significant departure from the preceding case is that  $Y_1$  declines less rapidly while  $Y_2$  registers a sharper increase (see Tables 4.5 and 4.6). As a consequence total income ( $Y_1 + Y_2$ ) increases in contrast to the decline shown in the preceding section. The level of income inequality as represented by  $Y_1/Y_2$  also declines more sharply with increasing values of  $\lambda$ . Thus, within a programme of consumption redistribution of the type defined by equations (4.44), growth of total income and reduction of income inequality emerge as complementary objectives.

The above result may not be intuitively obvious at first sight because under this scheme of redistribution total savings remain unchanged. Therefore a question could be posed as to how income ( $Y_1 + Y_2$ ) could increase beyond the level corresponding to the case without redistribution (i.e.  $\lambda = 0$ ). The explanation lies in the mechanism defined by the last equation of (4.45) according to which the entire amount retained by government goes into capital formation. This introduces an element into the system with a high growth rate of government capital  $K_g^I$ . The growth rate in a capital accumulation process of the type discussed here is the product of the savings rate and the output-capital ratio. Since the savings rate is unity,



at least for the portion of income transferred to government, the corresponding element of capital accumulation will have a much higher growth rate than those of other capital stocks in the system. Therefore the incomes of the rich and the poor would register corresponding increases by virtue of employment in government. However, the implicit assumption here is that the "Please" effect is negligible. Please {1967} has pointed out that although the government would be more farsighted than the individual who has a natural preference for present consumption, pressures on government to spend on consumption tend to become irresistible particularly when funds are available.

#### 4.10 Consequences of Population Increase

Recent investigations have brought into focus some of the interrelationships that exist between population change and income distribution. For example Rich {1973} who had carried out an analysis of the links between per capita income of the lowest 60 per cent of income earners and birth rates in 40 countries concluded that income accruing to the poorest groups contribute more to fertility reduction than increase of average per capita income as a whole. Again, the World Bank Study {1974 ; pp. 147-148} including 64 countries concluded that:

"..... each additional percentage point of total income received by the poorest 40 per cent reduces fertility by 2.9 points. By contrast, each additional year of life expectancy at birth, reduces the fertility index by 1.86 points..... The coefficients suggest that fertility decline is much

more sensitive to changes in income at the bottom end of the distribution."

Conversely, it is self evident that a fertility decline amongst the poor could lead to a reduction of income inequality in per capita terms, within growth cum redistribution processes of the type discussed in this study. For, lower the rate of growth of population amongst the poor, higher would be their per capita income and lower would be the extent of income inequality.

It would be beyond the scope of this study to examine in detail the interrelationships between population change and income inequality. Instead in this section we shall examine the consequences of fertility differentials as between the rich and the poor upon the growth of their per capita incomes. For this purpose we shall consider the growth cum redistribution processes discussed in the two preceding section. The time horizon will be taken as 10 years.

In section 4.2, the rich were defined as those belonging to the richest 20 per cent of the population. That is, the initial rich to poor ratio was taken as 1:4. Taking the former as the unit of population and denoting the rich and poor population by  $N_1$  and  $N_2$  respectively.

$$N_1 = 1 \quad \text{and} \quad N_2 = 4 \quad \text{at} \quad t = 0.$$

The overall rate of population growth has fluctuated between 1.6

and 1.7 per cent in recent years.<sup>1</sup> However, there are no estimates of fertility differentials in relation to income groups.<sup>2</sup> In these circumstances, as in the case of the Chenery-Ahluwalia model {1974; p.216}, we shall assume a lower rate of population growth for the rich and a higher rate for the poor. The justification for this assumption is the evidence (referred to above) that, fertility declines with higher levels of income. Representing the exponential rate of growth of population of the rich by  $r_1$  and that of the poor by  $r_2$  we shall consider the following estimates initially:

$$r_1 = 0.015$$

$$r_2 = 0.018$$

As a first step, these rates will be assumed constant over the 10 year period to be considered, and we shall refer to this case as Variant I. Then, the respective populations in year  $t$  are given by:

$$\begin{aligned} N_1 &= e^{0.015t} \quad \text{and} \\ N_2 &= 4e^{0.018t} \quad \dots \quad (4.63) \end{aligned}$$

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<sup>1</sup> See Central Bank Annual Report 1975-77.

<sup>2</sup> See Department of Census & Statistics {1978}, which presents the results of the World Fertility Survey, Sri Lanka, 1975 (WFS 1975). Although several aspects relating to fertility have been covered, that relating to fertility differentials by income groups has not been covered.

Thus, at  $t = 0$ ,  $N_1 = 1$ ,  $N_2 = 4$  and

at  $t = 10$ ,  $N_1 = 1.16$ ,  $N_2 = 4.80$

We shall next consider Variant II under which  $r_2$  is assumed to decrease progressively so that its average over the 10 year period is the same as that of  $r_1$  i.e. 0.015.

Thus, at  $t = 0$ ,  $N_1 = 1$ ,  $N_2 = 4$  and

at  $t = 10$ ,  $N_1 = 1.16$ ,  $N_2 = 1.16 \times 4 = 4.64$

As a third alternative, we shall also consider Variant III under which  $r_2$  is made to decrease more sharply so that its average value over the 10 year period reaches a level of 0.010

Then at  $t = 0$ ,  $N_1 = 1$ ,  $N_2 = 4$  and

at  $t = 10$ ,  $N_1 = 1.16$ ,  $N_2 = 4.42$

Let us define the incomes per unit of population by the following equations:

$$x_1 = \frac{Y_1}{N_1}$$

$$x_2 = \frac{Y_2}{N_2} \quad \dots \quad (4.64)$$

and take  $x_1/x_2$  as a measure of income inequality.

Under each variant described above we shall consider the following

cases of redistribution with growth.

Case A: Redistribution of Income with  $\lambda = 0.10$  (see Table 4.5)

Case B: " " " "  $\lambda = 0.25$  ( " " " )

Case C: " " Consumption with  $\lambda = 0.10$  (see Table 4.6)

Case D: " " " with  $\lambda = 0.25$  (see Table 4.6)

The results obtained in respect of each variant are set out in Tables 4.7 to 4.9.

Table 4.7

Changes in the values of  $x_1$  and  $x_2$  over time-Variant I

Case	$x_1$		$x_2$		$x_1/x_2$	
	t=0	t=10	t=0	t=10	t=0	t=10
A	38	50.59	13	14.17	2.92	3.57
B	38	47.07	13	14.85	2.92	3.17
C	38	50.85	13	14.34	2.92	3.54
D	38	47.68	13	15.39	2.92	3.10

Table 4.8

Changes in the values of  $x_1$  and  $x_2$  over time - Variant II

Case	$x_1$		$x_2$		$x_1/x_2$	
	t=0	t=10	t=0	t=10	t=0	t=10
A	38	50.59	13	14.66	2.92	3.45
B	38	47.07	13	15.36	2.92	3.06
C	38	50.85	13	14.83	2.92	3.43
D	38	47.68	13	15.92	2.92	2.99

Table 4.9Changes in the values of  $x_1$  and  $x_2$  over time - Variant III

Case	$x_1$		$x_2$		$x_1/x_2$	
	t=0	t=10	t=0	t=10	t=0	t=10
A	38	50.59	13	15.39	2.92	3.29
B	38	47.07	13	16.12	2.92	2.92
C	38	50.85	13	15.57	2.92	3.27
D	38	47.68	13	16.72	2.92	2.85

Results of Table 4.7 indicate the extent to which income inequality in per capita terms could deteriorate<sup>1</sup> despite strong redistributive measures, in circumstances where the population growth rate of the poor exceeds that of the rich. Even in the least unfavourable case i.e. Case D, income inequality changes from 2.92 to 3.10. But under Variant II, which assumes that the population growth rate of the poor could be maintained at the same level as of the rich, the extent of deterioration of income inequality is seen to be curbed to some extent. It is only under Variant III which is based on a substantial lowering of the rate of population growth of the poor that there appears to be possibilities of maintaining income inequality at the same level

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<sup>1</sup>This deterioration is traceable to the decline in the value of the parameter  $\beta_2$  from 0.049 in Set 1 to 0.028 in Table 4.4 resulting from <sup>2</sup>the change of the value of parameter  $s_2$  from 0.17 to 0.10 (see note at end of Table 4.4).

(Case B) and of a slight improvement in income inequality (Case D).

The above results highlight the inadequacy of redistributive measures unless they are coupled with measures to reduce population growth rates of the poor. Drastic reductions in population growth rate are not altogether impossible.<sup>1</sup> As pointed out in the WFS 1975 (see pp. 154-155) fertility is continuing to decline at a rapid rate - fast enough to halve fertility in 10 years. The decline is mainly attributed to the rise in mean age at marriage and secondly to the reduction in marital fertility. This report has also highlighted the prospects for<sup>a</sup> successful programme in fertility reduction.

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As noted in Chapter 2, the population growth rate which stood at 2.7 per cent per annum during the period 1953-63 had declined to 1.6 per cent per annum by 1974.

#### 4.11 Summary

We commenced the substantive part of our study by introducing in this chapter the basic form of the model for redistribution with growth. The Chenery - Ahluwalia model was extended by introducing the government as a separate entity whilst maintaining the basic features of the original model which characterize a developing economy. These features were a dualistic mode of production consisting of a modern sector which uses hired labour and a traditional sector based on self employment, concentration in the ownership of capital and differential savings rates for the rich and the poor.

The model showed how wage incomes are derived through employment linkages with the modern and government sectors as well as how profit and self employment incomes are derived. This was expressed concisely by showing how the distribution of incomes is determined by the wage and profitability matrix and the vector of capital stocks. We also showed that the model could be regarded as an extension of the Kaldor - Pasinetti model. In contrast to the Chenery - Ahluwalia model which had used simulation techniques, the model developed in this study was formulated in terms of a set of differential equations.

The empirical result with which we sought to test the validity of the model was the Kuznets curve according to which income inequality in a low income country has a tendency to trace an

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inverted "U" shape over time as the country develops. We demonstrated that, in general the model is capable of generating the Kuznets curve. A further elaboration of the model by the introduction of specific redistributive measures and plausible values for the relevant parameters enabled us to obtain as a limiting case, a situation where the Kuznets curve was indiscernible. We hypothesized that this limiting case should represent the Sri Lankan situation where the trends in the recent past has been one of a steady decline of income inequality due to specific redistributive measures.

As mentioned at the outset the principal aim of this study was to explore interrelationships that could exist between the processes of growth and redistribution of incomes by means of a formal model representative of the Sri Lankan situation. As a first step in this direction, we considered the static form of the model and posed a question regarding the type of capital that ought to be expanded if the incomes of the poor are to be improved. Subject to the assumption that the estimates of the parameters used were reasonably accurate, the result we obtained was that from the standpoint of improving the incomes of the poor, policies which stimulate increases in capital for self employment are preferable to those that expand state capital. The static form of the model also demonstrated that although nationalization of assets owned by the private sector may expand employment, it may not unambi-

quously reduce income inequality.

Dynamic considerations of the model yielded analytical solutions which appeared to be somewhat more complex than expected considering the relatively simple assumptions made at the outset. These solutions demonstrated the complex nature of the interrelationships that could exist in a situation of interdependent growth (as between income groups) implicit in the Chenery-Ahluwalia type of model.

The first important result that emerged from dynamic considerations of the model was that the rate of growth of incomes of the poor depends to a great extent upon the output-capital ratio relating to the capital owned by the poor and their marginal savings rate. This result highlighted the role of self reliance and the importance of improving the productivity of capital owned by the poor as well as raising their marginal savings rate.

More importantly, the analytical solutions obtained elaborated the manner in which the incomes of the poor are related to the activities of the modern (private) sector and to those of the government sector, within the dynamic framework. The second important result we obtained from dynamic considerations of the model concerns the income of the poor  $Y_2$ . It was shown that  $\frac{\partial Y_2}{\partial \alpha_1}$  was positive where  $\alpha_1$  was identified as the main parameter

associated with the growth of incomes of the rich, in a situation where the rich derive a major part of their income from the modern (private) sector rather than from the government. The policy implication derived was that slowing down the rate of growth of income of the rich would adversely affect the income of the poor, the underlying reason being the decrease in wage employment. Conversely, if the income of the poor are to be improved through employment, expansion of the modern (private) sector should receive high priority.

The third important area of investigations was brought about by the introduction of specific redistributive measures into the model. Results showed that while income inequality would decline under these circumstances, total income too would decline, the underlying reason being the diminution of total savings due to differential savings rates as between the rich and the poor. Thus, income redistribution and growth of total income emerged as competitive objectives.

We then proceeded to explore the alternative strategy of redistribution of consumption. That is, a strategy whereby the amount taxed from the rich is not entirely passed on to the poor, but only a proportion determined by the marginal consumption rates of the rich and the poor, the balance being retained by government for direct investment. This introduced in formal terms a system

of redistribution where there was no loss in total savings and the overall productivity of capital was raised by the special assumption we made about government savings and investment. The net result was that income inequality declined while total income increased. Thus, under a scheme of consumption redistribution, the redistributive and growth objectives emerged as complementary, subject to the assumption that the "Please" effect does not take place. This result may be regarded as the fourth important result we obtained from the dynamic considerations of the model.

Lastly, we set out to inquire what these growth cum redistribution processes mean in terms of per capita incomes. These investigations revealed that unless the rate of growth of population of the poorer segments of the community are drastically reduced, income inequality in per capita terms is bound to increase. These results highlighted the priority that ought to be attached to population control policies.

CHAPTER 5A MODEL FOR REDISTRIBUTION WITH GROWTH:  
SOME ELABORATIONS

"Can we conceive of the existence of a theory of economic growth which would neither be too closely tied to a particular historical situation nor resemble a game of entrepreneurial blindman's bluff, but would provide some relevant insights? - not a description of reality but (as Joan Robinson says) a device for sorting out our ideas?" T.W. Swan (Golden Ages and Production Functions)

### 5.1 An Extension of the Model

In the previous chapter, we had made several simplifying assumptions in order to demonstrate as clearly as possible the options available for a programme of redistribution with growth. In this chapter an attempt will be made to extend the model by incorporating several features so as to make it rather more closely representative of the real situation. The object is not to arrive at an exact representation of reality, but to take account of some of the more important features of the real situation and to evolve the model in a manner which will enable us to 'sort out our ideas', gain further insights and to discuss certain policy implications that are likely to emerge.

There are three important features which require to be introduced. They are:

- (a) direct and indirect taxes charged by government and a subsidy to the poor;
- (b) an institutional mechanism (a 'financial institution'), through which the rich can invest a part of their savings and from which the government borrows funds for development; and
- (c) a flow of foreign aid to finance government investment.

These features were selected particularly in view of their prominence in the formulation of national plans and annual budgets. The direct tax  $T_d$  is taken as a proportion of the income of the rich and is defined by the equation below:

$$T_d = t_d Y_1 \quad \dots \quad (5.1)$$

In reality, different tax rates are applicable to different income groups. But as it is beyond the scope of this model to go into such detail, we shall use only the average rate  $t_d$ .

Likewise, indirect taxes  $T_i$  too are conceived in terms of average rates and are assumed to be linked to outputs  $Q_1^l$  and  $Q_g^l$  by the equation:

$$T_i = t_{i1} Q_1^l + t_{ig} Q_g^l \quad \dots \quad (5.2)$$

Indirect taxes include business turnover taxes, export duties as the main components. Rather than showing these taxes as separate entities, for purposes of simplicity, these are being shown in aggregate form, the underlying assumption being that

they are all related to the output of the modern sector as well as to the output by the state sector. For, business turnover taxes, export duties and import duties are separately related to output and we could always work out weighted averages  $t_{i1}$  and  $t_{ig}$  relating to output by the modern (private) sector and the state sector respectively; specifically, a linear relationship is assumed. As for  $Q_1^n$ , the tax on it need not be taken into account in equation (5.2). Since  $Q_1^n$  accrues only to the rich by way of return on savings invested, it is akin to personal income. Accordingly, we shall take account of the tax on it through  $T_d$ .

The subsidy  $S$  is regarded as a transfer from government income to the income of the poor and is assumed to grow at a uniform rate of  $\epsilon$ , as given in the equation below:

$$S = S_0 e^{\epsilon t} \quad \dots \quad (5.3)$$

The system of financial institutions that exists in reality consisting of Banks, Finance Companies, the system of Treasury Bills, the Public Debt, etc., is too complex to be readily incorporated into the model. Further, savings are made both by the rich and the poor and are utilized by themselves as well as by government. We shall assume that the savings of the rich are diverted in the first instance towards augmenting  $K_1^l$  and that the balance savings of the rich are utilized by government through

rupee loans and Treasury Bills. That is, in contrast to the previous assumptions, we shall now assume that the 'non linked' capital stock  $K_1^n$  owned by the rich is the total amount lent by the rich to the government for investment. In the case of the poor, it is assumed that there are no 'excess' savings and that whatever savings are available are utilized for augmenting  $K_2^n$ .

$K_1^n$  can be regarded as representing the total net domestic public debt. The income that will accrue from  $K_1^n$  will be given by

$$Q_1^n = b_1 K_1^n \quad \dots \quad (5.4)$$

$Q_1^n$  may be regarded as the interest accruing from the investment of  $K_1^n$ .

It is assumed that the government will make use of the fund  $K_1^n$  to create real assets which will generate an output  $a_g K_1^n$ . Since  $b_1 K_1^n$  has to be paid out to the rich as interest, the net income to government will be  $(a_g - b_1) K_1^n$ . Thus, the total income of government will be given by the expression

$$\begin{aligned} & Q_g^l (1 - t_{1g}) + T_i + T_d + (a_g - b_1) K_1^n \\ &= Q_g^l (1 - t_{1g}) + t_{ig} Q_g^l + t_{i1} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \\ &= Q_g^l + t_{i1} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \quad \dots \quad (5.5) \end{aligned}$$

As seen above, the indirect tax component charged by government from government enterprises cancels out. This gross income may be divided into the wage components received by the two income



groups by virtue of employment in government which are

$$\begin{aligned} w_{1g} [Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n] \\ w_{2g} [Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n] \quad \dots \quad (5.6) \end{aligned}$$

and the component of income retained by the government for all other activities, namely

$$p_g [Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n] \quad \dots \quad (5.7)$$

$$\text{where } w_{1g} + w_{2g} + p_g = 1 \quad \dots \quad (5.8)$$

The expression (5.7) represents total government income net of personal emoluments, and we shall refer to this component as "government income". This restricted notion of government income is particularly relevant to the broad distributional question we are discussing. For, personal emoluments paid out are an integral part of the incomes of the rich and the poor, and the balance which remains could be regarded as the income of an "impersonal" government. It is this balance income that could be diverted to various other government activities depending on the policy options. As in the case of equations (4.2) of the previous chapter, we may write the wage equations of the rich and the poor as

$$\begin{aligned} w_1 &= w_{11} Q_1^l + w_{1g} [Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n] \\ w_2 &= w_{21} Q_1^l + w_{2g} [Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n] \quad \dots \quad (5.9) \end{aligned}$$

The profit incomes of the two income groups remain unchanged and are given, as before, by

$$\begin{aligned} P_1 &= p_1 Q_1^l + Q_1^n \\ P_2 &= Q_2^n \end{aligned} \quad \dots \quad (5.10)$$

But in the case of the government, the "profit income" or "government income" will now be given by

$$P_g = p_g \left[ Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \right] \dots \quad (5.11)$$

The parameters in the above equations are not altogether independent but are governed by the manner in which outputs are divided. The output  $Q_1^l$  is divided into  $w_{11} Q_1^l$ ,  $w_{21} Q_1^l$ ,  $p_1 Q_1^l$  and  $t_{11} Q_1^l$ . Therefore, it follows that

$$w_{11} + w_{21} + p_1 + t_{11} = 1 \quad \dots \quad (5.12)$$

Making the necessary adjustments for direct taxes on income of the rich, and a subsidy which is regarded as a transfer of income from government to the poor, the distribution of incomes will be determined by

$$\begin{aligned} Y_1 &= W_1 + P_1 - T_d \\ Y_2 &= W_2 + Q_2^n + S \\ Y_g &= P_g - S \end{aligned} \quad \dots \quad (5.13)$$

Substituting from equations (5.9), (5.10), (5.11), equations (5.13) read:

$$\begin{aligned}
Y_1 &= w_{11} Q_1^l + w_{1g} \left[ Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \right] \\
&\quad + p_1 Q_1^l + Q_1^n - T_d \\
Y_2 &= w_{21} Q_1^l + w_{2g} \left[ Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \right] \\
&\quad + Q_2^n + S \\
Y_g &= p_g \left[ Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \right] - S \dots \quad (5.14)
\end{aligned}$$

In order to check for consistency, when we add up equations (5.14) we get

$$\begin{aligned}
Y_1 + Y_2 + Y_g &= [w_{1g} + w_{2g} + p_g] \left[ Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \right] \\
&\quad + [w_{11} + w_{21} + p_1] Q_1^l - T_d + Q_2^n + Q_1^n \\
&= \left[ Q_g^l + t_{11} Q_1^l + t_d Y_1 + (a_g - b_1) K_1^n \right] + \\
&\quad [w_{11} + w_{21} + p_1] Q_1^l - T_d + Q_2^n + Q_1^n \\
&\quad \text{(by virtue of relation (5.8))} \\
&= Q_g^l + (a_g - b_1) K_1^n + (w_{11} + w_{21} + p_1 + t_{11}) Q_1^l \\
&\quad + t_d Y_1 - T_d + Q_2^n + Q_1^n \\
&= Q_g^l + Q_1^l + (a_g - b_1) K_1^n + Q_1^n + Q_2^n \\
&\quad \text{(by virtue of relation (5.8))} \\
&= Q_g^l + (a_g - b_1) K_1^n + (w_{11} + w_{21} + p_1 + t_{11}) Q_1^l \\
&\quad + t_d Y_1 - T_d + Q_2^n + Q_1^n \\
&= Q_g^l + Q_1^l + (a_g - b_1) K_1^n + Q_1^n + Q_2^n \\
&\quad \text{(by virtue of relation (5.12))} \\
&= Q_1^l + Q_g^l + a_g K_1^n + Q_2^n \dots \quad (5.15) \\
&\quad \text{(by virtue of relation (5.4))}
\end{aligned}$$

Thus, the total domestic income equates to the total domestic output (value added) by the rich ( $Q_1^l$ ), by the poor ( $Q_2^n$ ), and by the government ( $Q_g^l + a_g K_1^n$ ). Eliminating the  $Q$ s by the use of equations (4.1) and after a slight re-arrangement of terms we get:

$$\begin{aligned}
 (1-w_{1g} t_d) Y_1 &= (w_{11} + p_1) a_1 K_1^l + w_{1g} \left[ a_g K_g^l + a_1 t_{11} K_1^l \right. \\
 &\quad \left. + (a_g - b_1) K_1^n \right] + b_1 K_1^n - T_d \\
 Y_2 &= (w_{21} + w_{2g} t_{11}) a_1 K_1^l + w_{2g} \left[ a_g K_g^l + t_d Y_1 \right. \\
 &\quad \left. + (a_g - b_1) K_1^n \right] + b_2 K_2^n + S \\
 Y_g &= p_g \left[ a_g K_g^l + a_1 t_{11} K_1^l + t_d Y_1 + (a_g - b_1) K_1^n \right] - S \\
 &\quad \dots (5.16)
 \end{aligned}$$

The above equations may be regarded as extensions of equations (4.7). It is clear from the above equations that the distribution of incomes is determined not only by the distribution of capital stocks, wage and profitability parameters but also by the level of subsidies and the rates of direct and indirect taxes.

## 5.2 The Static Case

As in the case of equation (4.7), the picture presented by equation (5.16) is essentially a static one. Nevertheless there are several interesting policy implications contained in these equations. Some of the policy implications are basically the same as those discussed in Chapter 4 (soon after equation 4.7).

But since equations (5.16) are an extended form of equations (4.7) there are some important additional features that have emerged. Ignoring for a moment the difference in revenue implications, the second equation of (5.16) shows three major options available for raising the income of the poor i.e. expansion of State capital ( $K_g^1$ ), expansion of capital for self employment ( $K_2^n$ ) and increase of subsidies (S). The choice between the first and the second has already been discussed in Chapter 4. But the choice between the second and the third is perhaps more important. Let us assume that a subsidy of Rs.1000 m. purely for consumption is under consideration. An alternative proposal would be to grant Rs.500 million as a subsidy and Rs.500 million as a capital grant in the form of capital assets (eg. land, irrigation facilities or equipment) to augment  $K_2^n$  i.e. capital for self employment.<sup>1</sup> If we take  $b_2 = 0.30$  then the total addition to  $Y_2$  through the second proposal will be Rs.650m<sup>2</sup>, instead of Rs.1000m in terms of the first proposal. If the latter procedure is followed in second year too, then the new addition to  $Y_2$  would be Rs.800m<sup>3</sup>, as against Rs.1000m, in terms of the former. If the process is repeated

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<sup>1</sup> It should be noted that the distinction between a subsidy and a capital grant is not as clear cut as it would appear. For, it is possible for the recipient to save some part of the subsidy towards capital formation. On the other hand a capital grant could release some part of the recipient's funds for consumption which would have otherwise gone into saving.

<sup>2</sup> That is: Rs.500m + 0.3 x Rs.500m = Rs.650m.

<sup>3</sup> That is: Rs.500m + 0.3 x Rs.1000 = Rs.800m

during the third year too, then the addition to  $Y_2$  in the third year will be Rs.950m. In other words, by the end of the third year the addition to  $Y_2$  is around Rs.1000m, but the burden upon the government budget thereafter will only be Rs.500m in the form of a subsidy; the balance will be a recurring return to the poor from the capital increment of Rs.1500m built up over three years. The government would thus have additional funds at its disposal and could be preferably used for development. The numbers used are purely for purposes of illustration and several variants are possible. For example, if the drop during the first year from Rs.1000m to Rs.650m is considered to be sharp, then a more favourable alternative will be a subsidy of Rs.700m and a capital grant of Rs.300m, to yield an addition of Rs.790m to  $Y_2$ . The choice in favour of the latter option is further strengthened if we take account of the additional revenue generated through indirect taxes on increased output. Such revenue would obviously lower the burden upon the government budget.

Admittedly, the above discussion has been carried out in aggregate terms and one major drawback is the distributional problem that is bound to arise. For, a subsidy such as the rice subsidy is spread widely amongst the poor, whereas in the case of capital grants for self employment, only certain segments of the poor may be benefited. We shall return to this question later.

The third equation of (5.16) also serves to summarise the usual methods available for raising of government income. The first term within the square bracket represents the raising of government income through public investment, the second term represents indirect taxation, the third term represents direct taxation and the fourth term represents financing of government investments through borrowing. Thus, the third equation of (5.16) also expresses concisely the choices available to government for raising of revenue. For example, if direct taxation is to be reduced by  $\Delta(t_d Y_1)$  while holding government income constant, then one of the ways in which it could be compensated is by increasing the indirect tax rate  $t_{11}$  by  $\Delta(t_d Y_1)/a_1 K_1^I$ . Alternatively, it could be compensated by raising each of the other components as well so that the total increase equates to the loss of tax revenue. On the other hand if the subsidy  $S$  is to be lowered by  $\Delta S$  this equation shows the range of choices available for lowering direct and indirect taxes, while keeping  $Y_g$  constant.

Another source of funds available to government is foreign aid which we shall now take into account. We shall assume that there is a balance of payments deficit which is being met out of foreign aid. We shall consider only the net foreign aid component  $F_t$  (i.e. receipts less total debt service) and assume that this will be utilised by government for investment in

development projects. Taking into account the possibility of an improvement in the foreign exchange regime due to increases in domestic production (particularly agricultural production) we shall assume a moderate decrease of  $F_t$  over time given by the formula

$$F_t = F_0 e^{\delta t}, \quad \delta < 0 \quad \dots (5.17)$$

With the introduction of  $F_t$  for government investment<sup>1</sup> the savings-investment equations (4.15) will undergo a slight modification as follows:

$$\begin{aligned} \dot{K}_1^l &= q s_1 Y_1 \\ \dot{K}_1^n &= (1 - q) s_1 Y_1 \\ \dot{K}_2^n &= s_2 Y_2 \\ \dot{K}_g^l &= s_g Y_g + F_t \end{aligned} \quad \dots (5.18)$$

The parameters  $s_1$ ,  $s_2$ ,  $s_g$  and  $q$  retain the same meanings as in equation (4.15).

### 5.3 Dynamic Considerations

With a view to developing the model into a dynamic one we shall begin by differentiating equations (5.16) and rearranging terms to yield:

$$\begin{aligned} [1 + t_d (1 - w_{1g})] \dot{Y}_1 &= (w_{11} + w_{1g} t_{11} + p_1) a_1 \dot{K}_1^l + w_{1g} a_g \dot{K}_g^l \\ &\quad + [w_{1g} a_g + b_1 (1 - w_{1g})] \dot{K}_1^n \\ \dot{Y}_2 &= (w_{21} + w_{2g} t_{11}) a_1 \dot{K}_1^l + w_{2g} a_g \dot{K}_g^l \end{aligned}$$



$$\begin{aligned}
& + w_{2g} t_d \dot{Y}_1 + w_{2g} (a_g - b) \dot{K}_1^n + b_2 \dot{K}_2^n + \epsilon s_0 e^{\epsilon t} \\
\dot{Y}_g = & p_g a_g \dot{K}_g^l + p_1 t_{i1} a_1 \dot{K}_1^l + p_g t_d \dot{Y}_1 \\
& + p_g (a_g - b_1) \dot{K}_1^n - \epsilon s e^{\epsilon t} \quad \dots (5.19)
\end{aligned}$$

Eliminating the  $\dot{K}$ s using equations (5.18) and by further rearrangement of terms we get

$$\begin{aligned}
[1 + t_d (1 - w_{1g})] \dot{Y}_1 = & [(w_{11} + w_{1g} t_{i1} + p_1) a_1 q s_1 \\
& + (1 - q) \{w_{1g} a_g + b_1 (1 - w_{1g})\} s_1] Y_1 \\
& + w_{1g} a_g (s_g Y_g + F_t) \\
\dot{Y}_2 = & [(w_{21} + w_{2g} t_{i1}) a_1 q s_1 + (a_g - b_1) w_{2g} \\
& (1 - q) s_1] Y_1 \\
& + w_{2g} t_d \dot{Y}_1 + b_2 s_0 \\
& + b_2 s_2 Y_2 + w_{2g} a_g s_g Y_g + w_{2g} a_g F_t \\
& + \epsilon s_0 e^{\epsilon t} \\
\dot{Y}_g = & p_g a_g s_g Y_g + [p_1 t_{i1} a_1 q s_1 \\
& + p_g (a_g - b_1) (1 - q) s_1] Y_1 + p_g a_g F_t \\
& + p_g t_d \dot{Y}_1 - \epsilon s e^{\epsilon t} \quad \dots (5.20)
\end{aligned}$$

These equations are of the form

$$\begin{aligned}
\dot{Y}_1 = & \alpha_1 Y_1 + \gamma_1 Y_g + \eta_1 F_t \\
\dot{Y}_2 = & \beta_0 + \alpha_2 Y_1 + \beta_2 Y_2 + \gamma_2 Y_g + \eta_2 F_t + \nu_2 \dot{Y}_1 + \epsilon s_t \\
\dot{Y}_g = & \alpha_3 Y_1 + \gamma_3 Y_g + \eta_3 F_t + \nu_3 \dot{Y}_1 - \epsilon s_t \quad \dots (5.21)
\end{aligned}$$

$$\text{where } \alpha_1 = \frac{(w_{11} + w_{1g} t_{11} + p_1) a_1 q s_1 + (1-q) \{w_{1g} a_g + b_1 (1-w_{1g})\} s_1}{1 + t_d (1-w_{1g})}$$

$$\gamma_1 = \frac{w_{1g} a_g s_g}{1 + t_d (1-w_{1g})}$$

$$\eta_1 = \frac{w_{1g} a_g}{1 + t_d (1-w_{1g})}$$

$$\alpha_2 = [(w_{21} + w_{2g} t_{11}) a_1 q + (a_g - b_1) w_{2g} (1-q)] s_1$$

$$\beta_0 = b_2 s_0$$

$$\beta_2 = b_2 s_2$$

$$\gamma_2 = w_{2g} a_g s_g$$

$$\eta_2 = w_{2g} a_g$$

$$\gamma_2 = w_{2g} t_d$$

$$\alpha_3 = p_1 t_{11} a_1 q s_1 + p_g (a_g - b_1) (1 - q) s_1$$

$$\gamma_3 = p_g a_g s_g$$

$$\eta_3 = p_g a_g$$

$$\gamma_3 = p_g t_d$$

... (5.22)

Equations (5.21) may be regarded as an extension of equations (4.20). For, if we suppress the additional features by setting  $F_t = 0$ ,  $t_d = 0$ ,  $S_t = 0$ ,  $t_{11} = 0$ , and ignore the financial institutions we get back the set of equations (4.20). Before

proceeding to solve the equations (5.21), as in the case of Chapter 4, some interesting observations could be made. In order to obtain these let us divide the second equation of (5.21) throughout by  $Y_2$  to get

$$\frac{\dot{Y}_2}{Y_2} = \frac{\beta_0}{Y_2} + \alpha_2 \frac{Y_1}{Y_2} + \beta_2 + \gamma_2 \frac{Y_E}{Y_2} + \eta_2 \frac{F_t}{Y_2} + \nu_2 \frac{\dot{Y}_1}{Y_2} + \epsilon \frac{S_t}{Y_2} \dots \quad (5.23)$$

Comments made soon after equation (4.22) are applicable to the above equations too. But one of the new features in equation (5.23) is the manner in which foreign aid contributes to the rate of growth of the incomes of the poor. In like manner,  $F_t$  will contribute to the rate of growth of income of the rich too. But the contribution to the former will be greater provided that

$$\frac{\eta_2}{Y_2} > \frac{\eta_1}{Y_1} \quad \dots \quad (5.24)$$

$$\text{i.e. } \frac{w_{2E} a_E}{Y_2} > \frac{w_{1E} a_E}{[1 + t_d (1 - w_{1E})] Y_1}$$

$$\text{or } \frac{w_{2E}}{Y_2} > \frac{w_{1E}}{[1 + t_d (1 - w_{1E})] Y_1} \quad \dots \quad (5.25)$$

It is clear from the numerical values used in Chapter 4 (see Set 1 or Set 2) that  $\frac{w_{2E}}{Y_2} > \frac{w_{1E}}{Y_1}$ . Since the factor  $[1 + t_d (1 - w_{1E})] > 1$ , it follows that inequality (5.25) will hold. The implication of this is that  $F_t$  will contribute more towards the rate of growth

of incomes of the poor than that of the rich. Again, equation (5.23) also shows that the rate of growth of income of the rich contributes to the rate of growth of incomes of the poor. However the impact of this term is reduced by the factor  $\gamma_2 = w_{2g} t_d$  which is the product of the wage parameter  $w_{2g}$  and the direct tax rate  $t_d$ . The origin of this feature could be traced back to the second of equations (5.16) according to which direct taxes from the rich contribute to government revenue from which the poor derive a wage income through employment in government. The policy implication is that curbing the rate of growth of income of the rich will adversely affect the rate of growth of income of the poor.

The other obvious comment we could make regarding the equation (5.23) is that the rate of growth of incomes of the poor also depends upon the rate of growth of the subsidy too, as represented by the term  $\epsilon S_t$ . Although this is true, it did appear from discussion carried out a little earlier, that mechanisms which bring about a growth of capital stock for self employment is a better alternative. A further point is that the term  $\epsilon S_t$  adversely affects  $\dot{Y}_g$ , the full implications of which could be seen only after obtaining a complete solution to the set of equations (5.21) to which we shall proceed now.

The complete solution to the system of equations (5.21) is given by the set of equations below (see Appendix VII for solution)

$$\begin{aligned}
 Y_1 &= A e^{\lambda_1 t} + B e^{\lambda_2 t} + \mu_1 F_0 e^{\delta t} - \mu S_0 e^{\epsilon t} \\
 Y_g &= \frac{1}{\gamma_1} \left[ (\lambda_1 - \alpha_1) A e^{\lambda_1 t} + (\lambda_2 - \alpha_1) B e^{\lambda_2 t} + (\delta - \alpha_1) \mu_1 F_0 e^{\delta t} \right. \\
 &\quad \left. - (\epsilon - \alpha_1) \mu_2 S_0 e^{\epsilon t} \right] \\
 Y_2 &= C e^{\beta_2 t} - \frac{\beta_0}{\beta_2} + (\alpha_2 + \gamma_2 \alpha_1) \left[ \frac{A e^{\lambda_1 t}}{(\lambda_1 - \beta_2)} + \frac{B e^{\lambda_2 t}}{(\lambda_2 - \beta_2)} \right. \\
 &\quad \left. + \frac{\mu_1 F_0 e^{\delta t}}{(\delta - \beta_2)} - \frac{\mu_2 S_0 e^{\epsilon t}}{(\epsilon - \beta_2)} \right] \\
 &\quad + \frac{(\gamma_2 + \gamma_2 \gamma_1)}{\gamma_1} \left[ \frac{(\lambda_1 - \alpha_1) A e^{\lambda_1 t}}{(\lambda_1 - \beta_2)} + \frac{(\lambda_2 - \alpha_1) B e^{\lambda_2 t}}{(\lambda_2 - \beta_2)} \right. \\
 &\quad \left. + \frac{\{(\delta - \alpha_1) \mu_1 - \eta_1\} F_0 e^{\delta t}}{(\delta - \beta_2)} - \frac{(\epsilon - \alpha_1) \mu_2 S_0 e^{\epsilon t}}{(\epsilon - \beta_2)} \right] \\
 &\quad + (\eta_2 + \gamma_2 \eta_1) \frac{F_0 e^{\delta t}}{(\delta - \beta_2)} \\
 &\quad + \frac{\epsilon S_0 e^{\epsilon t}}{(\epsilon - \beta_2)} \quad \dots (5.26)
 \end{aligned}$$

where the constants A, B and C are given by

$$A = \frac{\gamma_1 \bar{Y}_E - (\lambda_2 - \alpha_1) \bar{Y}_1 + [\eta_1 - (\delta - \lambda_2) \mu_1] F_0 + (\epsilon - \lambda_2) \mu_2 S_0}{(\lambda_1 - \lambda_2)}$$

$$B = \frac{(\lambda_1 - \alpha_1) \bar{Y}_1 - \gamma_1 \bar{Y}_E + \mu_2 S_0 (\lambda_1 - \epsilon) + F_0 [\mu_1 (\delta - \lambda_1) - \eta_1]}{(\lambda_1 - \lambda_2)}$$

$$\begin{aligned} \text{and } C = & \bar{Y}_2 + \frac{\beta_0}{\beta_2} - (\alpha_2 + \nu_2 \alpha_1) \left[ \frac{A}{\lambda_1 - \beta_2} + \frac{B}{\lambda_2 - \beta_2} + \frac{\mu_1 F_0}{\delta - \beta_2} - \frac{\mu_2 S_0}{\epsilon - \beta_2} \right] \\ & - \frac{(\gamma_2 + \nu_1 \gamma_1)}{\gamma_1} \left[ \frac{(\lambda_1 - \alpha_1) A}{(\lambda_1 - \beta_2)} + \frac{(\lambda_2 - \alpha_1) B}{(\lambda_2 - \beta_2)} + \frac{\{(\delta - \alpha_1) \mu_2 - \eta_1\} F_0}{(\delta - \beta_2)} \right. \\ & \left. - \frac{(\epsilon - \alpha_1) \mu_2 S_0}{(\epsilon - \beta_2)} \right] - (\eta_2 + \nu_2 \eta_1) \frac{F_0}{(\delta - \beta_2)} - \frac{\epsilon S_0}{\epsilon - \beta_2} \end{aligned}$$

$$\text{and } \lambda_1 = \frac{\alpha_1 + \gamma_1 + \gamma_1 \nu_3 + \sqrt{(\alpha_1 + \gamma_3 + \gamma_1 \nu_3)^2 - 4(\alpha_1 \gamma_3 - \alpha_3 \gamma_1)}}{2}$$

$$\lambda_2 = \frac{\alpha_1 + \gamma_1 + \gamma_1 \nu_3 - \sqrt{(\alpha_1 + \gamma_3 + \gamma_1 \nu_3)^2 - 4(\alpha_1 \gamma_3 - \alpha_3 \gamma_1)}}{2}$$

$$\mu_1 = \frac{\eta_1 (\delta - \gamma_3) + \eta_3 \gamma_1}{\delta^2 - (\alpha_1 + \gamma_3 + \gamma_1 \nu_3) \delta + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)}$$

$$\mu_2 = \frac{\epsilon \gamma_1}{\epsilon^2 - (\alpha_1 + \gamma_3 + \gamma_1 \nu_3) \epsilon + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)} \quad \dots (5.27)$$

The above equations illustrate that interdependencies can be of a somewhat complex nature in a dynamic growth cum redistribution system when we take into account factors such as direct taxation, indirect taxation, subsidies, government (domestic) borrowing and foreign aid. But reality is indeed more complex, and had we incorporated further factors in an attempt to make the model a closer representation of reality, the results obtained would have been far more intractable. The above solution for  $Y_2$  is inevitably more complex in comparison with those for  $Y_1$  and  $Y_g$  because of its direct linkages with  $Y_1$  through wage income and subsidies.

The solution for  $Y_2$  may be regarded as a generalization of equation (4.27) of the previous chapter. As in the case of equation (4.27), it is clear from the term  $Ce^{\beta_2 t}$  in the solution for  $Y_2$  in the above equations that the growth of incomes of the poor depends in the first instance, upon the parameter  $\beta_2$  which is the product of the output-capital ratio and the marginal savings rate of the poor. As noted previously, this implies that the poor will have to save and invest more in order to generate greater output, or alternatively raise output per unit of capital employed. The policy implication of this is that efforts at stimulating savings amongst the poor and at enabling them to invest more in order to increase their stock of capital should receive high priority. Efforts in these directions have been already made in Sri Lanka and in Chapter 9 we shall discuss

the possibilities of stepping up such activities.

Since we have incorporated several new features into the model, one would expect more interpretations from equation (5.25) than from equation (4.27) relating to the simple model. This is in fact the case; for example, the direct impact of the subsidy upon  $Y_2$  is represented by the term

$$\frac{\epsilon S_0 e^{\epsilon t}}{\epsilon - \beta_2}$$

Suppose for instance  $\beta_2 = 0$  then this term reduces to  $S_0 e^{\epsilon t}$  which is the total subsidy at time  $t$ . This shows that in a situation where  $\beta_2$  is near zero, that growth of incomes of the poor will largely depend upon the growth of subsidies. When  $\beta_2 \neq 0$  the total subsidy  $S_0 e^{\epsilon t}$  gets altered by a factor  $\frac{\epsilon}{\epsilon - \beta_2}$ . There is no discontinuity at  $\epsilon = \beta_2$ . As in the case of the  $\beta_2$  argument following equation (4.25) it could be shown by considering the term  $\frac{\epsilon S_0 e^{\epsilon t}}{\epsilon - \beta_2}$  together with the term  $\frac{-\epsilon S_0 e^{\beta_2 t}}{\epsilon - \beta_2}$  that the solution exists when  $\epsilon \rightarrow \beta_2$ .

While the term  $\frac{\epsilon S_0 e^{\epsilon t}}{\epsilon - \beta_2}$  reflects the direct effect of subsidies, the sum of the other two terms in  $e^{\epsilon t}$  namely:

$$- \frac{\epsilon [(\gamma_2 + \gamma_2 \gamma_1) + (\alpha_2 \gamma_1 - \alpha_1 \gamma_2)] \mu_2 S_0 e^{\epsilon t}}{\gamma_1 (\epsilon - \beta_2)} = I \text{ (say)}$$



in equation (5.26) reflects the indirect effect of the subsidy. The origin of those two terms could be traced back to the effect of subsidies or government income.

$$\text{Since } \mu_2 = \frac{\epsilon \gamma_1}{\epsilon^2 - (\alpha_3 + \gamma_1 + \gamma_1 \gamma_3)\epsilon + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)}$$

$$I = \frac{-[\epsilon(\gamma_2 + \gamma_2 \gamma_1) + (\alpha_2 \gamma_1 - \alpha_1 \gamma_2)]\epsilon S_0 e^{\epsilon t}}{(\epsilon - \beta_2) [\epsilon^2 - (\alpha_1 + \gamma_3 + \gamma_1 \gamma_3)\epsilon + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)]} \dots (5.30)$$

Clearly as  $\epsilon$  increases,  $I$  decreases. Thus, if the rate of growth of the subsidy is higher, there will be a greater diminution in growth of government income, which in turn would lead to a diminution in the growth of wage incomes of the poor.

Similar discussion can be carried out regarding the direct and indirect effects of foreign aid, direct taxes and indirect taxes. But the usefulness of the model could perhaps be better appreciated if numerical values are assigned to the parameters and growth patterns are analysed. In particular we could analyse the time paths of  $Y_1$ ,  $Y_2$  and  $Y_g$  under alternative values of parameters, as for example alternative values for the rate of growth of subsidies. The results of such an exercise could be of some help in deciding between the policy options available.

For purposes of illustration, four variants will be discussed.

We shall begin with the values of the parameters shown as Set 2 in the previous chapter except for the values of the  $\bar{Y}$  s. The  $\bar{Y}$  s have to be adjusted since they now represent post tax incomes. The direct tax rate will be taken as 0.1, since on the basis of 1972 data on income and on personal tax, the <sup>average</sup> rate works out to around 0.11. The indirect tax rate  $t_{i1}$  is far more difficult to be estimated. In the absence of a specific method, particularly the contribution to output from the modern sector, an estimate of  $t_{i1} = 0.5$  was thought to be a plausible value to work with. The rate of decline of foreign aid in real terms is taken as 1 per cent per annum. Initially, the rate of increases of subsidies was taken as 1 per cent.

The subsidy considered here has been limited to the food subsidy only, estimated at Rs.526m in 1972, representing 3.86 per cent of GNP.<sup>1</sup> For the purpose of this exercise the value taken was  $S_0 = 4$ . In the case of foreign aid its various forms such as Project Aid, Suppliers Aid, Suppliers Credit, Grants, Special Drawing Rights that came within the meaning of foreign aid discussed here came to about Rs.600m (net of repayment). Accordingly  $F_0$  was taken as 5.

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<sup>1</sup> See Mahalingasivam {1978; p.78}

Variant I

$$a_1 = 0.30 \quad b_1 = 0.32 \quad b_2 = 0.30 \quad a_g = 0.25$$

$$w_{11} = 0.1 \quad w_{21} = 0.4 \quad p_1 = 0.5$$

$$w_{1g} = 0.2 \quad w_{2g} = 0.6 \quad p_g = 0.2 \quad q = 0.6$$

$$s_1 = 0.15 \quad s_0 = -4 \quad s_2 = 0.18 \quad s_g = 0.10$$

$$t_d = 0.1 \quad t_{i1} = 0.5$$

$$\bar{Y}_1 = 34 \quad \bar{Y}_2 = 56 \quad \bar{Y}_g = 10$$

$$\delta = -0.01 \quad \epsilon = 0.01 \quad S_0 = 4 \quad F_0 = 5$$

Substitution of these values in equation (5.22) yield

$$\alpha_1 = 0.033 \quad \gamma_1 = 0.0046 \quad \eta_1 = 0.046$$

$$\alpha_2 = 0.0164 \quad \beta_2 = 0.054 \quad \beta_0 = -1.2$$

$$\gamma_2 = 0.015 \quad \eta_2 = 0.15 \quad \nu_2 = 0.06$$

$$\alpha_3 = 0.006 \quad \gamma_3 = 0.005 \quad \eta_3 = 0.05 \quad \nu_3 = 0.02$$

When these values are in turn substituted in the appropriate equations in Appendix VII we get

$$A = 40.06 \quad B = -3.59 \quad \mu_1 = -0.75 \quad \mu_2 = -0.32$$

$$\lambda_1 = 0.034 \quad \lambda_2 = 0.004$$

With these values, the solution (5.26) reads

$$Y_1 = 40.06 e^{0.034t} - 3.59 e^{0.004t} - 3.75 e^{-0.01t} + 1.28 e^{0.01t}$$

$$Y_g = 8.69 e^{0.034t} + 22.61 e^{0.004t} - 14.96 e^{-0.01t} - 6.3 e^{0.01t}$$

$$Y_2 = 22.22 + 89.4 e^{0.054t} - 43.50 e^{0.034t} - 5.58 e^{0.004t} - 7.2 e^{-0.01t} + 0.743 e^{0.01t}$$

At  $t = 10$

$$Y_1 = 50.55 \quad Y_2 = 103.27 \quad Y_g = 15.19$$

### Variant II

Let us assume that subsidies are held constant, i.e.  $\epsilon = 0$

Then the parameters will be the same as in Variant I except for:

$$\epsilon = 0 \quad \mu_2 = 0$$

$$A = 40.32 \quad B = -2.57 \quad C = 88.83$$

$$Y_1 = 40.32 e^{0.034t} - 2.57 e^{0.004t} - 3.75 e^{-0.01t}$$

$$Y_g = 8.76 e^{0.034t} + 16.2 e^{0.004t} - 14.96 e^{-0.01t}$$

$$Y_2 = 22.22 + 88.83 e^{0.054t} - 43.80 e^{0.034t}$$

$$- 3.97 e^{0.004t} - 7.28 e^{-0.01t}$$

At  $t = 10$

$$Y_1 = 50.57 \quad Y_2 = 102.7 \quad Y_g = 15.56$$

### Variant III

Let us consider the case of  $\epsilon = 0.05$

Then all the parameters will be the same as in Variant I

except for:

$$\epsilon = 0.05$$

$$\mu_2 = 0.312$$

$$A = 42.23$$

$$B = -3.23$$

$$C = 118.6$$

In this case, at  $t = 10$ , we get

$$Y_1 = 50.51$$

$$Y_2 = 105.51$$

$$Y_g = 12.5$$

#### Variant IV

Instead of increasing  $\epsilon$  let us keep it at  $\epsilon = 0.01$  and increase  $S_2$  to 0.2 so that  $\beta_2 = 0.06$ . Then all parameters are the same as in Variant I except for:

$$\beta_2 = 0.06$$

$$C = 72.26$$

In this case, at  $t = 10$ , we get

$$Y_1 = 50.55$$

$$Y_2 = 113.63$$

$$Y_g = 15.19$$

The results obtained may be summarised as follows:

Table 5.1

#### Summary of the Results of Variants I - IV

Income Share	Initial Values	Values of variables at $t = 10$			
		Variant I	Variant II	Variant III	Variant IV
$Y_1$	34	50.55 (29.1)	50.57 (30)	50.51 (30.0)	50.55 (28.2)
$Y_2$	56	103.27 (61.1)	102.7 (61)	105.51 (62.6)	113.62 (63.3)
$Y_g$	10	15.19 (9.0)	15.56 (9)	12.5 (7.4)	15.19 (8.5)
Total	100	169.01 (100%)	168.83 (100%)	168.52 (100%)	179.36 (100%)

The above results highlight the limited nature of the impact of subsidies upon the incomes of the poor at a terminal date. Bearing in mind that the estimates of  $S$  and other parameters reasonably reflect the Sri Lankan situation it would appear that, when  $\epsilon$  representing the rate of growth of subsidies is increased from 1 per cent to 5 per cent per annum, the income of the poor  $Y_2$  at  $t = 10$  changes only from 103.27 to 105.51. That is, a five fold increase in the rate of growth of subsidies bringing about only a marginal gain.

On the other hand if the parameter  $\beta_2$ , is slightly increased from 0.054 to 0.060, holding  $\epsilon$  constant at 0.01,  $Y_2$  increases from 103.27 to 113.62. These results indicate that a strategy which designed to improve the relative position of the poor within a growth cum redistributive process, should be primarily aimed at raising the growth rate of the incomes of the poor. That is policy measures should be designed to increase  $\beta_2$  which in the main parameter associated with the growth of incomes of the poor. Since  $\beta_2$  is the product of the output-capital ratio  $b_2$  and the marginal savings rate of the poor  $s_2$ , it follows that an increase in either parameters could bring about the desired result. The parameter  $b_2$  could be increased by a wide range of measures such as the provision of better irrigation facilities to farmers, distribution of high yielding seed varieties to farmers, provisions of equipment (eg.

farm mechanization) or promotion of rural industry. Alternatively, a package of measures such as higher interest rates on savings deposits, extension of branch banking, increasing the supply of investment goods could stimulate greater savings. We shall discuss these policies in greater detail in Chapter 9.

#### 5.4 Summary

An attempt was made in this chapter to bring the model developed in Chapter 4 closer to the real situation by incorporating three important elements. These were direct and indirect taxes, a financial institution, and the flow of foreign aid, all of which feature prominently in the formulation of national plans and annual budgets. We were thus able to show concisely that distribution of incomes is determined not only by the distribution of capital stocks, wage and profitability parameters, but also by the levels of direct and indirect taxes.

Static considerations of the model showed the advantages of moving away from subsidies in the direction of augmenting the capital stock of the poor. A numerical exercise carried out showed that although there would be a drop in the incomes of the poor initially, after a short period of time its initial value would be restored and would increase thereafter. More importantly, the numerical exercise showed the manner in which subsidy burden upon the

government budget could be halved and indicated the more favourable option of diverting the balance resources for development.

Likewise, dynamic considerations of the model yielded a number of results in addition to those obtained in Chapter 4. The role of foreign aid presented an interesting result. It was possible to demonstrate that foreign aid would contribute more towards the growth of incomes of the poor than of the rich. Another interesting result obtained was that the rate of growth of income of the rich contributes positively to that of the poor, the policy implication being that curbing the rate of growth of incomes of the rich would adversely affect the rate of growth of income of the poor. We noted that this result was traceable to tax revenue which enables the government to provide a higher wage income to the poor.

The complete solution showed that interdependencies amongst the various elements of the model could be somewhat more complex in a dynamic growth cum redistribution process when we take account of additional features mentioned above. The more generalised solution obtained reiterated the conclusion in Chapter 4 that the poor would have to save more and invest more in order to generate greater output or alternatively raise output per unit of capital employed and indicated the areas of priority. Distinctions were made between the direct and indirect effects of subsidies. We noted that the net result of a higher growth rate in subsidies would be a greater diminution in the growth of government income,



which in turn would lead to a diminution in the growth of wage incomes of the poor. We elaborated this point further in terms of numerical simulations which yielded somewhat more dramatic results. The entire exercise demonstrated that significant increases in the rate of growth of subsidies leads only to a marginal increase in the income of the poor at a <sup>terminal</sup> date. On the other hand the exercise demonstrated the manner in which the income of the poor at a terminal date could be increased through a marginal increase in the parameter  $\beta_2$  through measures that could either increase the productivity of capital owned by the poor or their marginal savings rate. We also noted that the former could include measures such as the provision of better irrigation facilities to farmers, farm mechanization, and that the latter could include measures such as higher interest rates on savings deposits or a greater supply of investment goods.

CHAPTER 6THE INCENTIVE EFFECT

"Economic development has led to a large secular decline in the work week ... Consequently the allocation and efficiency of non-working time may now be more important to economic welfare than that of working time; yet the attention paid by economists to the latter dwarfs any paid to the former" - G.S. Becker  
(A Theory of the Allocation of Time)

## 6.1 Introduction

The implementation of redistributive policies whether it be through direct taxation or through other measures discussed in this study is likely to alter the relative status of skilled manpower or the professional class in society. The professional class consisting of scientists, engineers, doctors, economists, planners, accountants, managers, administrators, entrepreneurs, and other skilled personnel, though relatively small in a developing country plays a dominant role in the development process of the country. Professionals are either at the helm of affairs, formulating economic policy and development plans or in charge of setting up new industrial ventures and guiding them through initial stages or engaged in other development work.

In Sri Lanka, the number of professional technical and related personnel with graduate or equivalent qualifications was estimated

to be around 21,300<sup>1</sup> in 1971 representing less than 0.5 per cent of the total labour force in 1971 estimated to be of the order of 4.5 million.<sup>2</sup> The crucial role of skilled manpower in the development process was referred to thus by the Cabinet Committee {1974} :

"We consider that it is necessary to recognise the pivotal role of scientific, professional and technical personnel in economic development. Their skills are vital to our national well being ..... our task is to develop a practical framework of policy which would ensure that optimum use is made of our scientific, professional and technical personnel".

It is in the spirit of this train of thought that we shall attempt to take account of skilled manpower in the model developed, at least in a formal sense and to the extent possible.

## 6.2 An Extension of the Model

In the discussion by Codippily {1974} attention was focussed only upon the process of trading off leisure for work by the explicit use of utility functions. The utility functions were conceived of as depending upon the two variables, leisure and income.

Although the process of trading off leisure for work, incentive

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<sup>1</sup> See Sessional Paper No. X of 1974 Report of the Cabinet Committee Inquiring into the Problem of Technologically Professionally and Academically Qualified Personnel leaving Sri Lanka, p.10.

<sup>2</sup> See, Department of Census and Statistics, Census of Population 1971, Sri Lanka, General Report, Colombo, 1978, p.124

losses and gains and the consequent impact upon total income based on wage rates were discussed in analytical terms, a major inadequacy was the absence of explicit production functions. We shall therefore redefine the production functions already introduced as being dependent upon skilled manpower as well as capital.

To begin with, let us go back to equations (5.14) and make certain simplifying assumptions such as ignoring indirect taxes, government borrowing the use of non linked capital and use simpler notions of government income for the sake of clarity of exposition. We could then write  $Y_1$  and  $Y_2$  as<sup>1</sup>

$$Y_1 = (w_{11} + p_1) Q_1(K_1, L_1) + w_{1g} Q_g(K_g, L_g) - t_d Y_1$$

$$Y_2 = w_{21} Q_1(K_1, L_1) + w_{2g} Q_g(K_g, L_g) + Q_2 + S \quad \dots (6.1)$$

where  $L_1$  denoted skilled manpower in the modern (private) sector and  $L_g$  denoted skilled manpower in the government sector.

Let us assume that the direct tax rate  $t_d$  is one of factors upon which  $L_1$  and  $L_g$  are dependent. Then, as  $t_d$  increases, one would expect  $L_1$  and  $L_g$  to decrease on account of trading off

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<sup>1</sup> Our previous assumptions about fixed shares of wages and profits would be less tenable in terms of the variable proportions implicit in this model.

leisure for work as shown by Codippily <sup>1</sup> {1974}. Then, for a small increase in  $t_d$  denoted by  $\Delta t_d$ , the changes in  $Y_1$  and  $Y_2$  denoted by  $\Delta Y_1$  and  $\Delta Y_2$  will be as follows:

$$\Delta Y_1 = (w_{11} + p_1) \frac{\partial Q_1}{\partial L_1} \frac{\partial L_1}{\partial t_d} \Delta t_d + w_{1g} \frac{\partial Q_g}{\partial L_g} \frac{\partial L_g}{\partial t_d} \Delta t_d - \Delta t_d Y_1 - t_d \Delta Y_1$$

$$\Delta Y_2 = \left[ w_{21} \frac{\partial Q_1}{\partial L_1} \frac{\partial L_1}{\partial t_d} + w_{2g} \frac{\partial Q_g}{\partial L_g} \frac{\partial L_g}{\partial t_d} \right] \Delta t_d \quad \dots (6.2)$$

In the above equations, the terms  $\frac{\partial Q_1}{\partial L_1}$  and  $\frac{\partial Q_g}{\partial L_g}$  are easily identified as the marginal products of skilled personnel in the private and

<sup>1</sup> Utility of the rich had been defined as

$$U_1 = U_1 (E_1, Y_1)$$

where  $E_1$  = leisure enjoyed and

$Y_1$  = earned income (profit income had been ignored)

Suppose  $H_1$  = maximum number of hours that could be worked  
(with zero leisure)

$w_1$  = actual number of hours worked

$r_1$  = hourly earning rate

$$\text{Then } U_1 = U_1 [H_1 - w_1, r_1 w_1]$$

Maximisation of  $U_1$  yields  $w_1$  as a function of  $r_1$

i.e.  $w_1 = f(r_1)$  and hence  $\Delta w_1 = f'(r_1) \Delta r_1$

Therefore  $\Delta Y_1 = \Delta(r_1 w_1)$

$$\begin{aligned} &= \Delta r_1 w_1 + r_1 \Delta w_1 \\ &= [w_1 + r_1 f'(r_1)] \Delta r_1 \end{aligned}$$

Clearly if  $\Delta r_1$  is negative  $\Delta Y_1$  will be negative since the term within square bracket is positive, assuming that  $f'(x) > 0$ .

government sectors. The terms  $\frac{\partial L_1}{\partial t_d}$  and  $\frac{\partial L_g}{\partial t_d}$  are the rates of change in the supply of  $L_1$  and  $L_g$  with respect to  $t_d$ . On account of the leisure - income preference relationship (see foot note on previous page) we should expect  $L_1$  and  $L_g$  to decrease with increases in  $t_d$ . That is

$$\begin{aligned} \frac{\partial L_1}{\partial t_d} &< 0 \\ \frac{\partial L_g}{\partial t_d} &< 0 \end{aligned} \quad \dots (6.3)$$

Since  $w_{21}$  and  $w_{2g}$  are positive and since the marginal products  $\frac{\partial Q_1}{\partial L_1}$  and  $\frac{\partial Q_g}{\partial L_g}$  are also positive it follows from equations (6.2) that  $\Delta Y_2 < 0$  for a positive value of  $t_d$ . That is, an increase in tax would have the immediate effect of lowering the income of the poor, the underlying reason being the decrease in the supply of skilled manpower in the private and government sectors and the consequent loss of output, on the assumption of course that the production functions are 'well behaved'. Supposing the tax derived is transferred to the poor, the net effect could be investigated as follows:

From the first of equations (6.1) it follows that:

$$Y_1 = \frac{(w_{11} + p_1) Q_1 + w_{1g} Q_g}{(1 + t_d)} \quad \dots (6.4)$$

Therefore the increase in tax obtained from the increment  $\Delta t_d$  will be:

$$\begin{aligned}
 Y_1 \Delta t_d &= \frac{(w_{11} + p_1) Q_1 + w_{1g} Q_g}{(1 + t_d)} \Delta t_d \\
 &= [(w_{11} + p_1) Q_1 + w_{1g} Q_g] [1 - t_d] \Delta t_d \quad \dots (6.5)
 \end{aligned}$$

If the increase in tax revenue so obtained is transferred to the poor then the net change of income of the poor will be given by:

$$\begin{aligned}
 Y_1 \Delta t_d + \Delta Y_2 &= [(w_{11} + p_1) Q_1 + w_{1g} Q_g] [1 - t_d] \Delta t_d \\
 &+ \left[ w_{21} \frac{\partial Q_1}{\partial L_1} \frac{\partial L_1}{\partial t_d} + w_{2g} \frac{\partial Q_g}{\partial L_g} \frac{\partial L_g}{\partial t_d} \right] \Delta t_d \\
 &= \left\{ [(w_{11} + p_1) Q_1 + w_{1g} Q_g] (1 - t_d) \right. \\
 &\left. + \left[ w_{21} \frac{\partial Q_1}{\partial L_1} \frac{\partial L_1}{\partial t_d} + w_{2g} \frac{\partial Q_g}{\partial L_g} \frac{\partial L_g}{\partial t_d} \right] \right\} \Delta t_d \quad \dots (6.6)
 \end{aligned}$$

The first term within square brackets represents the post-tax income of the rich. The second term represents the wage parameters of the poor multiplied by the respective marginal products of skilled manpower in the private and government sectors as well as the rate of change of their supply in relation to the tax rate. In view of the obvious difficulties in estimating these functions, particularly the rate of change of supply of skilled manpower with respect to the direct tax rate, we do not propose to quantify equation (6.6). The only conclusion we could make is that equation (6.6) sums up the positive and negative effects upon the income of

<sup>1</sup> More precisely, increase in tax =  $\frac{1}{2} [Y_1 + Y_1 - \Delta t_d] \Delta t_d \approx Y_1 \Delta t_d$

of the poor arising from an increase of direct taxes. The first term within the square brackets is positive while the second term is negative since  $\frac{\partial L_1}{\partial t_d}$  and  $\frac{\partial L_R}{\partial t_d}$  are negative. Therefore the net result would depend upon the relative magnitude of the two terms. There is room for the net result to be negative. In such a situation, it would appear that the very process of trying to improve the incomes of the poor by direct taxation and redistribution would be self defeating.

The above discussions are based upon static considerations of the model. Let us now shift our attention to the more important aspect of the problem i.e. that connected with possible decreases in skilled manpower over time. This would necessarily involve dynamic considerations of the model.

### 6.3 Dynamic Considerations

A decrease in the supply of skilled manpower can take place due to a variety of factors other than direct taxes referred to in the previous section. Perhaps the dominant factor amongst these is the "brain drain" caused by "pull" factors such as better opportunities for advancement abroad or shortages in manpower in certain fields or "push" factors such as low salaries, lack of recognition of merit, relatively low status of professionals in



the administrative system or a lack of incentives for professional advancement.<sup>1</sup> It is not possible to take account of these factors in analytical terms. But it is possible to indicate a general framework, in terms of the model developed, within which we can discuss the consequences of a decline in skilled manpower due to a variety of factors in general. Such a framework could be developed by differentiating equations (6.1) and an analogous equation for  $Y_g$  to obtain.

$$\begin{aligned}\dot{Y}_1 &= \frac{(w_{11} + p_1)}{(1 + t_d)} \left[ \frac{\partial Q_1}{\partial K_1} \dot{K}_1 + \frac{\partial Q_1}{\partial L_1} \dot{L}_1 \right] + \frac{w_{1g}}{(1 + t_d)} \left[ \frac{\partial Q_g}{\partial K_g} \dot{K}_g + \frac{\partial Q_g}{\partial L_g} \dot{L}_g \right] \\ \dot{Y}_2 &= w_{21} \left[ \frac{\partial Q_1}{\partial K_1} \dot{K}_1 + \frac{\partial Q_1}{\partial L_1} \dot{L}_1 \right] + w_{2g} \left[ \frac{\partial Q_g}{\partial K_g} \dot{K}_g + \frac{\partial Q_g}{\partial L_g} \dot{L}_g \right] \\ &\quad + b_2 \dot{K}_2 + \epsilon S_0 e^{\epsilon t} \\ &\quad (\text{since } Q_2 = b_2 K_2 \text{ and } S = S_0 e^{\epsilon t}) \\ \dot{Y}_g &= p_g \left[ \frac{\partial Q_g}{\partial K_g} \dot{K}_g + \frac{\partial Q_g}{\partial L_g} \dot{L}_g + t_d \dot{Y}_1 \right] \quad \dots (6.7)\end{aligned}$$

Subject to the simplifications mentioned above, the capital accumulation equations (5.18) would reduce to:

$$\begin{aligned}\dot{K}_1 &= s_1 Y_1 \\ \dot{K}_2 &= s_2 Y_2 \\ \dot{K}_g &= s_g Y_g + F_t \quad \dots (6.8)\end{aligned}$$

<sup>1</sup> See The Colombo Plan - Special Topic Papers {1972; pp. 1-3}

Substituting for  $\dot{K}_1^l$ ,  $\dot{K}_2^n$  and  $\dot{K}_g^l$  in equations (6.7) we get

$$\begin{aligned}\dot{Y}_1 &= \frac{(w_{11}+p_1)}{(1+t_d)} \left[ \frac{\partial Q_1}{\partial K_1} s_1 Y_1 + \frac{\partial Q_1}{\partial L_1} \dot{L}_1 \right] + \frac{w_{1g}}{(1+t_d)} \left[ \frac{\partial Q_g}{\partial K_g} (s_g Y_g + F_t) + \frac{\partial Q_g}{\partial L_g} \dot{L}_g \right] \\ \dot{Y}_2 &= w_{21} \left[ \frac{\partial Q_1}{\partial K_1} s_1 Y_1 + \frac{\partial Q_1}{\partial L_1} \dot{L}_1 \right] + w_{2g} \left[ \frac{\partial Q_g}{\partial K_g} (s_g Y_g + F_t) + \frac{\partial Q_g}{\partial L_g} \dot{L}_g \right] \\ &\quad + b_2 \dot{K}_2 + \epsilon S_0 e^{\epsilon t} \\ \dot{Y}_g &= p_g \left[ \frac{\partial Q_g}{\partial K_g} (s_g Y_g + F_t) + \frac{\partial Q_g}{\partial L_g} \dot{L}_g + t_d \dot{Y}_1 \right] \quad \dots (6.9)\end{aligned}$$

It is clear from the above equations that negative values of  $\dot{L}_1$  and  $\dot{L}_g$  representing decreases in the stock of skilled manpower in the private and government sectors would have a negative impact upon the growth rates of income both of the rich and the poor. That is, the net result would be a slowing down of the entire growth process. We shall not attempt to quantify these results on account of the extreme difficulties involved in estimating some of the terms.

The policy implication we could derive from these equations is the importance that should be attached to curbing the loss of skilled manpower. In the Sri Lankan context some remedial measures have been already adopted, such as exempting government employees from

income tax in respect of salaries received from government (see Budget Speech of November 1978) or the recent increase in the salaries of engineers. Efforts in these directions may require to be pursued further.

CHAPTER 7CONSIDERATIONS OF OPTIMAL GROWTH

"Consumption - to repeat the obvious - is the sole end of object of economic activity ... consumption for which we can profitably provide in advance cannot be pushed indefinitely into the future"  
J.M. Keynes (General Theory, page 104)

## 7.1 Introduction

Although a number of issues relating to income redistribution and economic growth have been discussed in the preceding chapters, a question we have not dealt with so far concerns optimality in a growth cum redistribution process. The basic question to be posed is, how can we choose an optimum path when there are several paths available for redistribution with growth? This necessarily involves the specification of an objective function and the optimisation of this function over time, within the framework of a growth cum redistribution process.

The central question posed in Ramsey's seminal paper of 1928 was "how much should a society save if welfare is to be optimised?" Literature on optimal economic growth initiated by this paper is so voluminous today, particularly due to its growth during the last two decades, that it seems beyond the scope of this study to make an adequate review. Instead it may suffice to note that considerations of optimal economic growth have been applied to a wide range of situations. Notable contributions include papers by Goodwin {1961} concerning optimal growth in an

underdeveloped economy, by Chakravarty {1962} on optimal savings within a finite time horizon, by Chakravarty {1965} again, on optimal capital accumulation in a multi-sector economy, by Uzawa {1964} and Srinivasan {1964} on optimal growth in a two sector economy, by Karl Shell {1967} on questions of optimality in capital accumulation when there is exogenous technical change, by Dixit {1968} on optimal development in a labour surplus economy, by Dixit {1969} again, on partial planning in a dual economy with particular reference to pricing policy for food, by Dorfman {1969} on the application of optimal control theory and by Iyoha {1972} on the formulation of the optimisation problem as a multi-stage decision problem under uncertainty, for obtaining a solution on a sequential basis with the use of dynamic programming techniques.

But apparently there has been no attempt to discuss optimal growth possibilities within a growth cum redistribution process of the type discussed in this study. Probably, the closest to it is the contribution by Hamada {1967} regarding the optimal transfer and income distribution in a growing economy. But he has discussed only the limited case where workers do not save at all. Moreover, the government is not taken account of as an explicit entity, and only one policy instrument is used.

In this chapter we shall make an attempt to incorporate optimal

growth considerations into the model developed in this study. But with a view to keeping the mathematical exposition in a manageable form we shall consider the application of optimal growth to a somewhat simplified form of the model.

## 7.2 The Model

In the simplified model used in this chapter, the outputs of the respective sectors are defined by the equations

$$\begin{aligned} Q_1 &= a_1 K_1 && \text{(Modern Sector)} \\ Q_2 &= \beta_2 K_2 && \text{(Traditional Sector)} \\ Q_g &= a_g K_g && \text{(Government Sector)} \end{aligned} \quad \dots (7.1)$$

where the  $Q$  s and the  $K$  s have the same notations as before and  $a_1$ ,  $\beta_2$ , and  $a_g$  are the output-capital ratios.

As before, it is assumed that the rich and the poor receive wage incomes from employment in the modern sector and that the former receive a profit income as well. The rest of the income of the poor consists of the output generated from capital owned by them and wage income from government. One simplifying assumption made here is that of ignoring the wage income of the rich from government. The model also has provision for a transfer of income to the poor from the incomes of the rich beyond a taxable limit  $Z_1$ . It is assumed that a proportion  $\lambda$  of the income of the rich beyond a level  $Z_1$  is transferred from the rich to the poor; in fact  $\lambda$  will be one of the control variables. Thus, the distribution of

incomes will then be determined according to the following equations.

$$\begin{aligned}
 \text{Income of the rich: } Y_1 &= Z_1 + (\alpha_1 K_1 - Z_1)(1 - \lambda) \\
 \text{Income of the poor: } Y_2 &= \alpha_2 K_1 + (\alpha_1 K_1 - Z_1)\lambda + \gamma_2 K_g + \gamma_3 K_g \\
 \text{Government Income: } Y_g &= \gamma_3 K_g \quad \dots (7.2)
 \end{aligned}$$

where  $a_1 = \alpha_1 + \alpha_2$ ,  $a_g = \gamma_2 + \gamma_3$ , and  $0 \leq \lambda \leq 1$

In the above equations  $\alpha_1 K_1$  represents the total income of the rich (from the modern sector) and it is assumed that this exceeds the taxable limit  $Z_1$ , i.e.  $(\alpha_1 K_1 - Z_1) > 0$ , so that there is a surplus available for transfer. But the quantum of such a transfer will depend on the value of  $\lambda$  which has to be determined. The term  $\alpha_2 K_1$  represents the wage income received by the poor from the modern sector and  $\gamma_2 K_g$  that received from the government sector. The term  $\gamma_3 K_g$  represents the government income net of personal emoluments. It is also assumed that the rich and the poor do not reverse roles during the time horizon considered (see Appendix IX regarding the validity of this assumption)

Assuming  $s_1$  to be the average savings rate of the rich, capital accumulation by the rich is defined by the equation:

$$\dot{K}_1 = s_1 [Z_1 + (1 - \lambda)(\alpha_1 K_1 - Z_1)] \quad \dots (7.3)$$

The savings rate  $s_1$  is assumed to be fixed;  $\dot{K}_1$  is the investment net of depreciation.

In the case of the poor it is assumed that wage income received

from government i.e.  $\gamma_2 K_g$ , wage income received from the modern sector i.e.  $\alpha_2 K_1$  as well as the income transferred i.e.  $\lambda(\alpha_1 K_1 - Z_1)$  are all consumed, but that a fixed proportion  $s_2$  of income derived from the use of their own capital is saved and invested.

In view of the importance attached to the growth of capital for self employment (see earlier chapters), we shall also make provision for the transfer of some part of government savings to the poor as capital grants to augment their stock of capital. Specifically, it is assumed that a proportion  $\mu$  of government savings beyond a level  $I_g$  is transferred to the poor in the form of capital grants. Investment  $I_g$  may be thought of as the average level of government investment that would have to be carried out in any case for "continuation projects" of government for which commitment has already been made. In other words, the implicit assumption is that  $I_g$  is exogenously defined. Accordingly, the capital accumulation equations of the poor and government are:

$$\begin{aligned}\dot{K}_2 &= s_2 \beta_2 K_2 + \mu (s_g \gamma_3 K_g - I_g) \\ \dot{K}_g &= I_g + (1 - \mu) (s_g \gamma_3 K_g - I_g) \quad \dots (7.4)\end{aligned}$$

where  $s_g$  is the average rate of savings by government assumed fixed, and  $0 \leq \mu \leq 1$ .



### 7.3 The Optimisation Problem

In Chapter 1 we briefly noted that the search for a "more or less ethics free" theory of welfare largely influenced by Pareto optimality had been a fruitless one. Subsequent approaches to welfare theory based on concepts of a social welfare function recognised that value judgements were inevitable if distributive questions were to be discussed. It is in the spirit of these developments that we make a value judgement here, that we are primarily concerned with the question of improving the welfare of the poor. Accordingly we shall consider the question of optimisation of the consumption of the poor, discounted over a finite period of time. The welfare of the rich is not altogether ignored; for, the very formulation of the model ensures that the rich will receive an income of  $Z_1$  at least and that transfers could take place only from their income in excess of  $Z_1$ . The usual practice is to consider the question of optimisation of utility of consumption rather than consumption itself. But with a view to obtaining explicit solutions and deriving clear cut policy guidelines we shall opt for the latter.<sup>1</sup> Such simplifications have been made, for example, by Uzawa {1964} , Karl Shell {1976} , Hamada {1967} Iyoha {1972} and by Heal {1973; pp. 299-301}.

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<sup>1</sup> The question of optimising utility of consumption will be considered in Section 7.5

The consumption function of the poor is given by

$$C = \alpha_2 K_1 + (\alpha_1 K_1 - Z_1)\lambda + (1 - s_2)\beta_2 K_2 + \gamma_2 K_g$$

Let  $\delta$  be the rate of discount, assumed to be positive.

We may now formulate the optimisation problem as:

$$\begin{aligned} & \text{Maximise} \int_0^T [\alpha_2 K_1 + (\alpha_1 K_1 - Z_1)\lambda + (1 - s_2)\beta_2 K_2 + \gamma_2 K_g] e^{-\delta t} dt \\ & \text{subject to} \\ & \quad (i) \text{ Constraints imposed by the capital accumulation} \\ & \quad \quad \text{equations (7.3) and (7.4)} \\ & \quad (ii) \dot{K}_1, \dot{K}_2, \dot{K}_g > 0 \\ & \quad (iii) K_1 = \bar{K}_1 > 0, K_2 = \bar{K}_2 > 0, K_g = \bar{K}_g > 0 \text{ at } t = 0 \\ & \quad (iv) 0 \leq \lambda \leq 1, 0 \leq \mu \leq 1 \end{aligned} \quad \dots (7.6)$$

and subject to the assumption that the rich and the poor do not reverse roles over the period  $(0, T)$  (see Appendix IX)

The two policy instruments in the hands of the government are represented by  $\lambda$  and  $\mu$ . That is, the government could control income transfers from the rich to the poor as well as the proportion of government savings transferred in the form of capital grants to the poor. The essence of the optimisation problem is to determine the manner in which  $\lambda$  and  $\mu$  should be varied in order to maximise the above integral subject to the constraints mentioned.

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<sup>1</sup> Alternatively one could take up the question of optimising per capita consumption of the poor rather than total consumption of the poor. In this case the change in the maximand essentially amounts to changing the exponential term. But the changes in the constraint equations could make the analysis more cumbersome.

Another important consideration that should be taken note of at this stage is that of terminal capital stocks. Although the optimisation exercise is carried out over a finite planning horizon, it does not mean that the post planning period could be altogether ignored. On the contrary the optimisation exercise must inter alia ensure that sufficient capital is left over at the end of the planning period under consideration so as to provide for output and consumption in the post planning period. One of the earliest attempts to take account of this problem in optimal growth literature was that of Chakravarty {1962}. He postulated that in order to provide for a capital stock which we would wish to bequeath for the future, the initial stock of capital  $K_0$  should grow at a rate  $g$  to a terminal value of  $K_0 e^{gt}$ . But the determination of a value for  $g$  was arbitrary, and consequently the determination of terminal capital stock remained arbitrary. The procedure followed later by Chakravarty {1969} sought to get over this problem by taking into account explicitly the rate of growth of consumption in the post planning period. He obtained an expression  $K(T) = C_T / (b-r)$  where  $b$  is the output - capital ratio,  $r$  is the rate of growth of consumption and  $C_T$  is the terminal year consumption which is left to be determined by the optimising mechanism. Again Sen {1967} suggested the notion of a "terminal margin" in terms of which an arbitrarily stipulated proportion of the terminal year's output should be saved. According to this

method the value judgement about how much of a margin should be left over is reduced to the choice of a value of  $\lambda$  in the interval (0,1).

Another approach to the question of determining terminal capital has been that of incorporating the terminal capital explicitly in the maximand. For example Manne {1974} considered the maximisation of the function

$$\sum_{t=1}^T u_t(c_t) + pk_{T+1}$$

where  $c_t$  = consumption at time  $t$ ,  
 $u_t$  = present value of cardinal utility of consumption  
 $k_{T+1}$  = terminal capital stock  
 $p$  = terminal stock valuation coefficient, assumed positive

The above function is maximised subject to a number of constraints following a linear programming approach rather than a Ramsey type one which we shall consider here. As in the case of a dynamic Leontief system, the dual variables are calculated recursively.

In the ensuing analysis (of his study) value judgements are found to be inevitable in determining terminal capital stocks. Depending on whether the planner is a conservative or a radical it is found that  $p = a^{T+1}$  or  $p = 0.35 a^{T+1}$  where  $a$  is the subjective discount factor.

Thus, the inevitability of value judgements and an element of arbitrariness in determining terminal capital stocks still

remain. This must necessarily be so. For, as well summed up by Heal {1973, pp.260-261} in order to determine how much the terminal capital stocks ought to be, the planning board must know:

- (i) how long after  $T$  the world will continue;
- (ii) what consumption possibilities from  $T$  onwards are implied by a given value of  $K_T$
- (iii) the form of preferences concerning post-plan consumption streams.

Reasons for arbitrariness in determining  $K_T$  are clearer now. For, no planning board could venture to make even guesses about (i) above. Nevertheless, some value judgements are possible in regard to (ii) and (iii) above and it is on the basis of these that one would have to proceed.

For the purpose of this study we shall make the assumption that terminal capital is determined by the optimisation process itself but that in per capita terms it will not be less than the initial capital. Assuming that the output-capital ratios remain unchanged, the above assumption ensures that output in per capita terms will not be below those prevailing at the beginning of the plan period. This assumption takes account of points (ii) and (iii) mentioned by Heal but without being too restrictive. For, per capita output levels are assured at a level not less than those prevailing initially, and actual consumption out of output in the post plan period could be left to be determined by the consumer preferences of this future

period. Our assumptions regarding terminal capital requirements may be spelt out algebraically as follows:

Let  $r_1$  = rate of population increase of the rich

$r_2$  = " " " " " " " poor

$$\bar{K}_1 = \bar{K}_1 e^{r_1 T}$$

$$\bar{K}_2 = \bar{K}_2 e^{r_2 T}$$

$$\bar{K}_g = \bar{K}_g$$

where  $\bar{K}_1$ ,  $\bar{K}_2$ ,  $\bar{K}_g$  represent the lower bounds of terminal capital or the minimum terminal capital required.

Then, according to our assumption concerning terminal capital,

$$\begin{aligned} K_1(T) &\geq \bar{K}_1 \\ K_2(T) &\geq \bar{K}_2 \\ K_g(T) &\geq \bar{K}_g \quad \dots \end{aligned} \quad (7.7)$$

the above inequalities complete the formulation of the optimisation problem.

#### 7.4 The Solution

Let the Hamiltonian be defined as

$$H = [\alpha_2 K_1 + (\alpha_1 K_1 - Z_1) \lambda + \gamma_2 K_g + (1 - s_2) \beta_2 K_2] + q_1 s_1 \{Z_1 + (1 - \lambda)(\alpha_1 K_1 - Z_1)\} \\ + q_2 \{s_2 \beta_2 K_2 + \mu(s_g \gamma_3 K_g - I_g)\} + q_3 \{I_g + (s_g \gamma_3 K_g - I_g)(1 - \mu)\} e^{-\delta t}$$

where  $q_1 e^{-\delta t}$ ,  $q_2 e^{-\delta t}$ ,  $q_3 e^{-\delta t}$  are the Hamiltonian multipliers.

$K_1$ ,  $K_2$ ,  $K_g$  are the state variables.

$q_1$ ,  $q_2$ ,  $q_3$  are the co-state variables

$\lambda$  and  $\mu$  are the control variables.

Applying Pontryagin's Maximum Principle {1962}, the conditions for an optimal solution are that there exists non-zero continuous functions  $q_1(t)$ ,  $q_2(t)$ ,  $q_3(t)$  within the interval  $0 \leq t \leq T$  such that

- (i) for each fixed set of  $K_1, K_2, K_g, q_1, q_2, q_3$ , a maximum of  $H(K_1, K_2, K_g, q_1, q_2, q_3, \lambda, \mu)$  is attained in the region defined by  $0 \leq \lambda \leq 1, 0 \leq \mu \leq 1$ .

$$(ii) \dot{q}_i - \delta q_i = - \frac{\partial H'}{\partial K_i}, \quad i = 1, 2, \text{ or } g \quad \dots (7.9)$$

where  $H'$  is the undiscounted Hamiltonian (i.e.  $H = H' e^{-\delta t}$ )

$$(iii) \dot{K}_i = \frac{\partial H'}{\partial q_i}, \quad K_i(0) = \bar{K}_i, \quad i = 1, 2, \text{ or } g \quad \dots (7.10)$$

and the transversality conditions.

$$(iv) q_i(T) e^{-\delta T} \geq 0, \quad q_i e^{-\delta T} [K_i(T) - \bar{K}_i] = 0 \quad \dots (7.11)$$

The co-state variables  $q_1, q_2, q_3$  may be interpreted as the shadow prices of capital accumulation by the rich, the poor and government in terms of consumption foregone by the poor in time  $t$ . The Hamiltonian  $H$  is of the form

$$H = [H_0 + (\alpha_1 K_1 - Z_1)(1 - q_1 s_1) \lambda + (s_g \gamma_3 K_g - I_g)(q_2 - q_3) \mu] e^{-\delta t} \quad (7.12)$$

where  $H_0$  is the sum of the terms independent of  $\lambda$  and  $\mu$ .

Since this function is linear in  $\lambda$  and  $\mu$  it follows that a maximum of  $H$  is attained at one of the corner points of the square  $0 \leq \lambda \leq 1, 0 \leq \mu \leq 1$ .

As could be seen from Figure 7.1 a maximum can be attained at A, B, C or D depending on

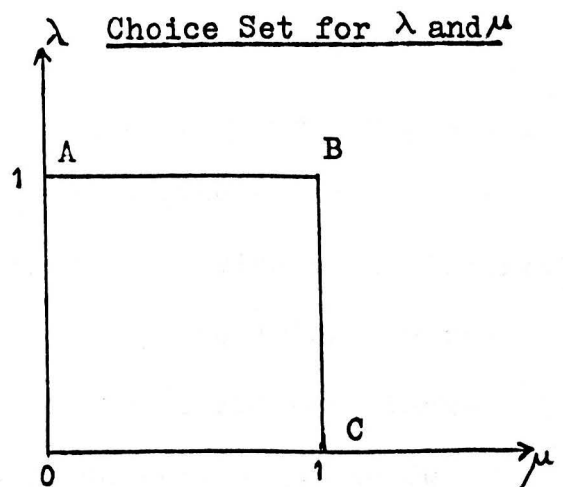


Figure 7.1

the signs of the coefficients of  $\lambda$  and  $\mu$ .

Since the terms  $(\alpha_1 K_1 - Z_1)$  and  $(s_g Y_3 K_g - I_g)$  were taken to be positive in the formulation of the model, it follows that the signs of the coefficients of  $\lambda$  and  $\mu$  will be entirely determined by  $(1 - q_1 s_1)$  and  $(q_2 - q_3)$ . Thus, there are four cases in all:

Case A : if  $1 - q_1 s_1 > 0$ ,  $q_2 - q_3 < 0$ , a maximum will be attained at A

Here  $\lambda = 1, \mu = 0$

Case B : if  $1 - q_1 s_1 > 0$ ,  $q_2 - q_3 > 0$  " " " " " B

Here  $\lambda = 1, \mu = 1$

Case C : if  $1 - q_1 s_1 < 0$ ,  $q_2 - q_3 > 0$  " " " " " C

Here  $\lambda = 0, \mu = 1$

Case D : if  $1 - q_1 s_1 < 0$ ,  $q_2 - q_3 < 0$  " " " " " D

Here  $\lambda = 0, \mu = 0$

That is, H could attain a maximum at any one of the points (0,1), (1,1), (1,0) or (0,0) and the solutions are of the "bang-bang" type.

Since the constants in the solution for  $q_1, q_2, q_3$  are determined by the boundary conditions at  $t = T$  and in view of the possibilities of switches from one case to another within the interval  $0 \leq t \leq T$ , we shall begin the analysis by inquiring how the system behaves in the neighbourhood of  $t = T$ , and work backwards thereafter. But the first question that ought to be settled is whether all cases are feasible at  $t = T$  ?.



From equation (7.3) it is clear that the lowest rate of accumulation of  $K_1$  would take place when  $\lambda = 1$ , in which case the equation would read as:

$$\dot{K}_1 = s_1 Z_1 \quad \dots \quad (7.13)$$

$$\text{Therefore } K_1 = K_1 + s_1 Z_1 t \quad \dots \quad (7.14)$$

$$\text{At } t = T, K_1(T) = \bar{K}_1 + s_1 Z_1 T$$

$$\text{But } \bar{K}_1 = \bar{K}_1 e^{r_1 T}$$

$$\text{Therefore } K_1(T) > \bar{K}_1,$$

$$\text{so long as } \bar{K}_1 + s_1 Z_1 T > \bar{K}_1 e^{r_1 T}$$

$$\text{or } s_1 Z_1 T > \bar{K}_1 (e^{r_1 T} - 1) \quad \dots \quad (7.15)$$

The above inequality is satisfied in terms of the data framework used (see Appendix VIII). It follows therefore that

$$[K_1(T) - \bar{K}_1] > 0 \quad \dots \quad (7.16)$$

which when taken together with the transversality conditions (7.11) yields

$$q_1(T) = 0 \quad \dots \quad (7.17)$$

That is, the shadow price of  $K_1$  is zero at  $t = T$  since the terminal capital requirement is over-fulfilled.

Similarly, it is clear from equation (7.4) that the lowest rate of accumulation of  $K_2$  would take place when  $\mu = 0$ , in which case the equation would read as:

$$\dot{K}_2 = s_2 \beta_2 K_2 \quad \dots \quad (7.18)$$

$$\therefore K_2 = \bar{K}_2 e^{s_2 \beta_2 t} \quad \dots \quad (7.19)$$

$$\text{But } \bar{K}_2 = \bar{K}_2 e^{r_2 T}$$

Therefore  $[K_2(T) - \bar{K}_2] > 0$

as long as  $s_2/\beta_2 > r_2$  ..... (7.20)

The above inequality too is satisfied by the parameters used  
(see Appendix VIII)

Therefore  $[K_2(T) - \bar{K}_2] > 0$  and it follows from the transversality conditions (7.11) that

$$q_2(T) = 0 \quad \dots \quad (7.21)$$

In the case of capital accumulation by government, it is clear that  $\dot{K}_g > 0$ . Since by definition  $\bar{K}_g = \bar{K}_g$ , it follows immediately that  $[K_g(T) - \bar{K}_g] > 0$ . Again, as above, the transversality conditions yield:

$$q_3(T) = 0 \quad \dots \quad (7.23)$$

Since  $q_1 = 0$  at  $t = T$  and  $q_1(t)$  is a continuous function of  $t$  within the interval  $(0, T)$ , the magnitude of  $q_1$  will be close to zero in the neighbourhood of  $t = T$ . In these circumstances the inequality  $1 - q_1 s_1 < 0$  cannot hold. This means that cases C and D are inadmissible at  $t = T$ .

From equations (7.9) the canonical equations for  $q_2$  and  $q_3$  are

$$\dot{q}_2 - \delta q_2 = - [(1-s_2)/\beta_2 + q_2 s_2/\beta_2]$$

$$\dot{q}_3 - \delta q_3 = - [\gamma_2 + q_2 s_g \gamma_3 \mu + q_3 s_g \gamma_3 (1-\mu)] \quad (7.24)$$

Therefore  $\dot{q}_2 - \dot{q}_3 = \gamma_2 - \beta_2(1-s_2) + \delta q_2 - q_2 s_2/\beta_2$

$$- \delta q_3 + q_2 s_g \gamma_3 \mu + q_3 s_g \gamma_3 (1-\mu) \quad (7.25)$$

Since  $q_2$  and  $q_3 \rightarrow 0$  as  $t \rightarrow T$ , it is clear from equation (7.25)

that  $(\dot{q}_2 - \dot{q}_3) \rightarrow \gamma_2 - \beta_2(1-s_2) \quad (7.26)$

Now  $\gamma_2 = 0.2$ ,  $\beta_2 = 0.28$ ,  $(1 - s_2) = 0.9$  (see Appendix VIII)

Therefore  $\gamma_2 - \beta_2 (1 - s_2) < 0$

i.e.  $(\dot{q}_2 - \dot{q}_3)$  tends to a negative value as  $t \rightarrow T$ .

Further,  $q_2 - q_3$  is zero at  $t = T$ .

Therefore  $q_2 - q_3$  must be positive just before  $t = T$ .

In these circumstances Case A is inadmissible.

at  $t = T$ . Thus, we are left with Case B as the only feasible case at  $t = T$ .

That is, irrespective of the case in which <sup>the</sup> system may begin or enter subsequently, the system must end in Case B.

Starting from this point we can now work backwards to find how the system behaves during  $0 \leq t \leq T$

The canonical equation for  $q_1$  is

$$\dot{q}_1 - \delta q_1 = -[\alpha_2 + \alpha_1 \lambda + q_1 s_1 \alpha_1 (1 - \lambda)] \quad \dots (7.27)$$

The relevant case in the neighbourhood of  $t = T$  is Case B.

Therefore  $\lambda = 1$ , and equation (7.27) reduces to

$$\dot{q}_1 - \delta q_1 = -(\alpha_2 + \alpha_1) = -a \quad \dots (7.28)$$

The general solution is

$$q_1 = Ae^{\delta t} + \frac{a}{\delta} \quad \dots (7.29)$$

where  $A$  is an arbitrary constant.

At  $t = T$ ,  $q_1 = 0$

$$\text{i.e. } Ae^{\delta t} + \frac{a}{\delta} = 0 \quad \dots (7.30)$$

$$\text{Therefore } q_1 = \frac{a}{\delta} [1 - e^{\delta(t-T)}] \quad \dots (7.31)$$

Clearly  $q_1$  is a continuous function of  $t$  for the range  $t$  defined and it steadily decreases to zero as  $t$  tends to  $T$ . This solution will hold as long as  $1 - q_1 s_1 > 0$  within the interval  $0 \leq t \leq T$ . That is, if we start from the point  $t = T$  and move backwards in time, the above will be the solution up to a point  $t_1 > 0$  at which point  $q_1 = \frac{1}{s_1}$ . Whilst this is the general case, it may well be that  $1 - q_1 s_1 > 0$  throughout the interval  $0 \leq t \leq T$ , in which case the above solution will hold throughout the interval  $0 \leq t \leq T$ . In this particular case  $\lambda = 1$  throughout the interval  $0 \leq t \leq T$ , implying that transfer of income from rich to the poor as defined by the equation (7.2) will take place throughout the plan period, with the parameter  $\lambda = 1$ .

In the general case just referred to, a switch can take place from one case to another, as shown below:

$$\text{At } t = t_1, q_1 = \frac{a}{\delta} [1 - e^{\delta(t_1 - T)}] = \frac{1}{s_1} \quad \dots \quad (7.32)$$

$$\text{Therefore } t_1 = T + \frac{1}{\delta} \log \left[ 1 - \frac{\delta}{as_1} \right] \quad \dots \quad (7.33)$$

$$\text{As a first approximation } t_1 = T + \frac{1}{\delta} \left[ -\frac{\delta}{as_1} - \frac{1}{2} \left( \frac{\delta}{as_1} \right)^2 \right] \quad (7.34)$$

---

<sup>1</sup> By Taylor's theorem  $\log_e(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$

where  $|x| < 1$

Thus, when a switch in the value of  $\lambda$  does take place within the interval  $0 \leq t \leq T$ , the above formulae enable us to calculate the corresponding point of time. For example if we assign the numerical values:-

$T = 25$  years,  $\delta = 0.05$ ,  $a_1 = 0.3$ ,  $s_1 = 0.25$   
equation (7.33) yields

$$t_1 = 3 \text{ years.}$$

That is, during the interval  $t_1 \leq t \leq T$ ,  $q_1(t)$  which is a continuous function of  $t$  steadily decreases from  $\frac{1}{s_1}$  to zero

as  $t$  increases from  $t_1$  to  $T$ . Throughout this interval  $1 - q_1 s_1 > 0$  and  $\lambda = 1$ . During this regime, capital accumulation takes place according to equations (7.3) and (7.4) with  $\lambda = 1$ .

In the general case, when a switch takes place as at time  $t_1$  as described above, it follows that just prior to  $t_1$ ,  $1 - q_1 s_1 < 0$  in which case  $\lambda = 0$ . The canonical equation for  $q_1$  then reads:

$$\begin{aligned} \dot{q}_1 - \delta q_1 &= -\alpha_2 - q_1 s_1 \alpha_1 \\ \text{i.e. } \dot{q}_1 - (\delta - s_1 \alpha_1) q_1 &= -\alpha_2 \end{aligned} \quad \dots \quad (7.35)$$

The general solution is

$$q_1 = A e^{(\delta - s_1 \alpha_1) t} + \frac{\alpha_2}{(\delta - s_1 \alpha_1)}$$

where  $A$  is an arbitrary constant.

$$\text{At } t = t_1, \quad q_1 = \frac{1}{s_1}$$

$$\text{i.e. } q_1 = A e^{(\delta - s_1 \alpha_1)t_1} + \frac{\alpha_2}{\delta - s_1 \alpha_1} = \frac{1}{s_1} \quad \dots (7.36)$$

$$\text{Therefore } A = \left[ \frac{1}{s_1} - \frac{\alpha_2}{\delta - s_1 \alpha_1} \right] e^{(\delta - s_1 \alpha_1)(-t_1)}$$

Thus, the complete solution is

$$q_1 = \left[ \frac{1}{s_1} - \frac{\alpha_2}{\delta - s_1 \alpha_1} \right] e^{(\delta - s_1 \alpha_1)(t - t_1)} + \frac{\alpha_2}{\delta - s_1 \alpha_1} \quad \dots (7.37)$$

It is assumed that  $\delta < a_1 s_1$ , which is a condition necessary for obtaining a value of  $t_1$  within  $(0, T)$ . This solution will hold as long as  $1 - q_1 s_1 < 0$  within the interval  $0 \leq t \leq T$ , that is within  $0 \leq t \leq t_1$ . Clearly  $q_1(t)$  as defined by equation (7.37) above is a continuous function of  $t$  and steadily decreases from

$$\left[ \frac{1}{s_1} - \frac{\alpha_2}{\delta - s_1 \alpha_1} \right] e^{(\delta - s_1 \alpha_1)(-t_1)} + \frac{\alpha_2}{\delta - s_1 \alpha_1} \text{ to } \frac{1}{s_1} \text{ as } t \text{ varies}$$

from 0 to  $t_1$ . During this regime capital accumulation takes place according to equations (7.3) and (7.4) with  $\lambda = 0$ .

Let us call the period  $(0, t_1)$  Regime I and period  $(t_1, T)$  Regime II (See figure 7.2)

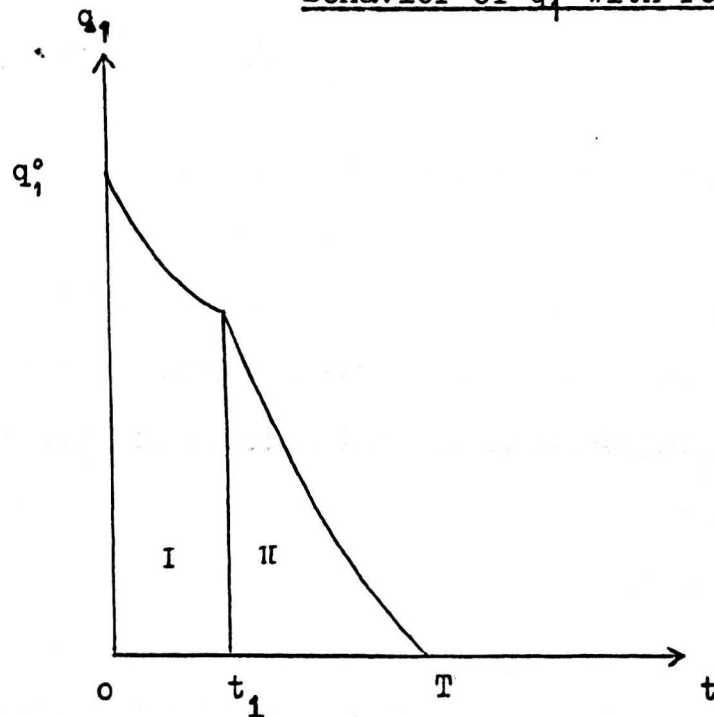
Behavior of  $q_1$  with respect to time

Figure 7.2

Then the policy implications are that an optimal growth cum redistribution programme calls for a liberal tax policy in Regime I. In fact  $\lambda = 0$  implies that there should be no taxes at all upon the incomes of the rich. Such situations are not uncommon in the real world. For, the real world example is the tax holiday i.e. a period of time over which enterprises are allowed to function and grow without taxation.<sup>1</sup> Again, this policy implication is also in accord with the contention mentioned earlier in this study that an economy must be allowed to grow before the fruits of economic growth could be redistributed. Thus the first phase may even increase (see Appendix X) income inequality before attempts are made to level off income inequality which again is the broad empirical

<sup>1</sup> See footnote on page 7-21.

pattern of the Kuznets hypothesis.

In Regime II,  $\lambda = 1$ . This indicates a strong measure of income redistribution. In fact the implication is that all income beyond a level  $Z_1$  should be transferred from the rich to the poor, and capital accumulation would take place according to equation (7.3) with  $\lambda = 1$ . This is admittedly an extreme case and is traceable to the form of the objective function, which is linear in  $\lambda$ . As could be seen later, it is possible to obtain an interior solution for  $\lambda$  i.e. an optimum solution where the value of  $\lambda$  will be between 0 and 1. In such a case the value of  $\lambda$  will indicate the optimum proportion of income (beyond  $Z_1$ ) that should be transferred from the rich to the poor.

In order to solve the problem completely we shall now proceed to investigate the behaviour patterns of  $q_2$  and  $q_3$ . It was shown earlier (see discussion soon after equation (7.26) that  $q_2 - q_3$  must be positive just prior to  $t = T$ . Since the relevant case just prior to  $t = T$  is Case B, it follows that  $\mu = 1$  just prior to  $t = T$ . If  $\mu = 1$ , the canonical equations for  $q_2$  and  $q_3$  read as:

$$\dot{q}_2 - \delta q_2 = - \left[ (1 - s_2) \beta_2 + q_2 s_2 \beta_2 \right] \quad \dots (7.38)$$

<sup>1</sup>For example, tax holidays up to a maximum of 5 years are offered to industries that are now being set up in the Investment Promotion Zone within the Greater Colombo Economic Commission. Similar concessions are also offered to Small and Medium Scale Industries which are set up outside Colombo.



$$\dot{q}_3 - \delta q_3 = - [\gamma_2 + q_2 s_g \gamma_3] \quad \dots (7.39)$$

Putting  $q_4 = q_2 - q_3$  we get

$$\dot{q}_4 - \delta q_4 = [\gamma_2 - (1 - s_2)/\beta_2] + q_2 (s_g \gamma_3 - s_2/\beta_2) \dots (7.40)$$

From equation (7.38)

$$\dot{q}_2 - (\delta - s_2 \beta_2) q_2 = -(1 - s_2)/\beta_2 \quad \dots (7.41)$$

$$\text{Therefore } q_2 = A e^{(\delta - s_2 \beta_2)t} + \frac{(1 - s_2)/\beta_2}{(\delta - s_2 \beta_2)} \quad \dots (7.42)$$

where A is an arbitrary constant

At  $t = T$ ,  $q_2 = 0$

$$\text{Therefore } A = - \frac{(1 - s_2)/\beta_2}{(\delta - s_2 \beta_2)} e^{(\delta - s_2 \beta_2)(-T)}$$

Therefore, the complete solution for  $q_2$  is

$$q_2 = \left[ 1 - e^{(\delta - s_2 \beta_2)(t - T)} \right] \frac{(1 - s_2)/\beta_2}{(\delta - s_2 \beta_2)}$$

Substituting for  $q_2$  in equation (7.40) we get

$$\dot{q}_4 - \delta q_4 = [\gamma_2 - (1 - s_2)/\beta_2] + G - G e^{(\delta - s_2 \beta_2)(t - T)} \quad (7.43)$$

$$\text{where } G = \frac{(s_g \gamma_3 - s_2/\beta_2)(1 - s_2)/\beta_2}{(\delta - s_2 \beta_2)} \quad \dots (7.44)$$

The complementary function =  $A e^{\delta t}$  and

$$\text{the particular integral} = \frac{1}{-\delta} [\gamma_2 - (1 - s_2)/\beta_2 + G] + \frac{G e^{(\delta - s_2 \beta_2)(t - T)}}{s_2 \beta_2} \dots (7.45)$$

Since  $q_2 = 0$  and  $q_3 = 0$  when  $t = T$ ,  $q_4 = 0$  when  $t = T$ .

$$\text{Therefore } A = \left[ \frac{1}{\delta} \{ \gamma_2 + (1 - s_2)\beta_2 + G \} - \frac{G}{s_2\beta_2} \right] e^{-\delta T} \quad (7.46)$$

Therefore the complete solution is

$$q_4 = \frac{1}{\delta} \left[ \gamma_2 - (1 - s_2)\beta_2 + G \right] \left[ e^{\delta(t-T)} - 1 \right] + \frac{G e^{\delta(t-T)}}{s_2\beta_2} \left[ e^{-s_2\beta_2(t-T)} - 1 \right] \quad (7.47)$$

Clearly  $q_4 = 0$  when  $t = T$ .

It is not immediately obvious whether or not  $q_4$  will be zero for other values of  $t$  within the interval  $(0, T)$ . These points of time (if any) could be found by solving the equation:

$$\frac{1}{\delta} \left[ \gamma_2 - (1 - s_2)\beta_2 + G \right] \left[ e^{\delta(t-T)} - 1 \right] + \frac{G e^{\delta(t-T)}}{s_2\beta_2} \left[ e^{-s_2\beta_2(t-T)} - 1 \right] = 0 \quad (7.48)$$

In order to investigate the possibility of obtaining roots of the equation (7.48) we shall consider the behaviour of  $q_4$  in the first instance. Differentiating equation (7.47) we get:

$$\dot{q}_4 = \left[ \gamma_2 - (1 - s_2)\beta_2 - \frac{G(\delta - s_2\beta_2)}{s_2\beta_2} \right] e^{\delta(t-T)} + G \frac{(\delta - s_2\beta_2)e^{(\delta - s_2\beta_2)(t-T)}}{s_2\beta_2} \dots (7.49)$$

Substituting for  $G$  from equation (7.44) we get,

$$\dot{q}_4 = \left[ \gamma_2 - s_g \gamma_3 \frac{(1 - s_2)}{s_2} \right] e^{\delta(t-T)} + (s_g \gamma_3 - s_2\beta_2) \frac{(1 - s_2)}{s_2} e^{(\delta - s_2\beta_2)(t-T)} \quad (7.50)$$

It could be shown that  $\dot{q}_4 < 0$  throughout the interval  $(0, T)$  as follows:

$$\text{Now } \gamma_2 < \beta_2 (1 - s_2) \quad (\text{see Appendix VIII}) \quad \dots \quad (7.51)$$

Subtracting the term  $\frac{s_g \gamma_3 (1 - s_2)}{s_2}$  from both sides we get

$$\gamma_2 - \frac{s_g \gamma_3 (1 - s_2)}{s_2} < \beta_2 (1 - s_2) - \frac{s_g \gamma_3 (1 - s_2)}{s_2} \quad (7.52)$$

For all  $t$  within the interval  $(0, T)$  and  $\delta > 0$

$$e^{\delta(T-t)} > e^{(\delta - s_2 \beta_2)(T-t)} \quad (7.53)$$

$$\text{Therefore } e^{\delta(t-T)} < e^{(\delta - s_2 \beta_2)(t-T)} \quad (7.54)$$

Multiplying inequalities (7.52)<sup>1</sup> and (7.54)<sup>2</sup> we get

$$\left[ \gamma_2 - \frac{s_g \gamma_3 (1 - s_2)}{s_2} \right] e^{\delta(t-T)} < \left[ s_2 \beta_2 (1 - s_2) - s_g \gamma_3 \right] \frac{(1 - s_2) e^{(\delta - s_2 \beta_2)(t-T)}}{s_2} \dots (7.55)$$

It follows therefore that  $\dot{q}_4 < 0$  throughout the interval  $(0, T)$

But  $q_4 = 0$  at  $t = T$ . Therefore  $q_4$  is positive throughout the interval  $(0, T)$ . That is,  $q_4$  is a steadily decreasing function of  $t$  within the interval  $(0, T)$  and it decreases to zero as  $t$  tends to  $T$ .

Since  $q_4 = q_2 - q_3 > 0$  throughout  $(0, T)$ ,

$\mu = 1$  throughout  $(0, T)$

The policy implication of this result is that all government savings beyond the level  $I_g$  ie. uncommitted government savings should be transferred to the poor as capital grants, if an optimal

1 L.H.S  $> 0$  See Appendix VIII

2 L.H.S  $> 0$  (Exponential function)

growth cum redistribution programme of the type specified here is to be carried out. This result also strengthens the arguments in the earlier chapters which highlighted the importance of increasing capital for self employment if the welfare of the poor is to be maximised within the plan period. As could be seen from Appendix VIII this result is traceable to the higher output-capital ratio associated with capital for self employment as compared with government capital and the comparatively higher share of output consumed. In these circumstances, capital accumulation takes place according to equations (7.4) with  $\mu = 1$  That is:

$$\dot{K}_2 = s_2 \beta_2 K_2 + (s_g \gamma_3 K_g - I_g) \quad \dots \quad (7.56)$$

$$\dot{K}_g = I_g \quad \dots \quad (7.57)$$

Since  $q_2 - q_3 > 0$  throughout the interval  $(0, T)$ , the only feasible cases are Case B and Case C. The choice between these two would depend on the sign of  $(1 - q_1 s_1)$ . As seen earlier  $(1 - q_1 s_1)$  could remain positive throughout the interval  $(0, T)$ , under which circumstances the only feasible case would be Case B. That is the system would start in Case B and end in Case B. But if  $(1 - q_1 s_1)$  does change sign within the interval  $(0, t)$ , it is negative (Regime I) and during the sub period  $(t_1, T)$  it is positive (Regime II), then the system will start in Case C, remain in Case C till  $t = t_1$  change over to Case B at  $t = t_1$  and remain in Case B throughout the period  $(t_1, T)$ .

We may now summarise the main results of this section: In the event of there being a switch from one regime to another during the plan period  $(0, T)$  i.e. a change over from Case C to Case B, the results are as follows:

### Recapitulation of Main Results

#### Regime I (Case C)

Sub period:  $0 \leq t \leq t_1$ , where  $t_1 = T + \frac{1}{\delta} \log \left( 1 - \frac{\delta}{a_1 s_1} \right)$

Control Variables:  $\lambda = 0$ ,  $\mu = 1$

Co-State Variables:  $q_1 = \left[ \frac{1}{s_1} - \frac{\alpha_2}{\delta - s_1 \alpha_1} \right] e^{(\delta - s_1 \alpha_1)(t - t_1)} + \frac{\alpha_2}{\delta - s_1 \alpha_1}$

$$q_4 = q_2 - q_3 = \frac{1}{\delta} \left[ \gamma_2 - (1 - s_2) \beta_2 + G \right] \left[ e^{\delta(t - \tau)} - 1 \right] + \frac{G e^{\delta(t - \tau)}}{s_2 \beta_2} \left[ e^{-s_2 \beta_2 (t - \tau)} - 1 \right]$$

These equations represent the manner in which the shadow price of capital accumulation would behave over the sub-plan period  $(0, t_1)$

State Variables: The capital accumulation equations are

$$\dot{K}_1 = s_1 \alpha_1 K_1$$

$$\dot{K}_2 = s_2 \beta_2 K_2 + (s_g \gamma_3 K_g - I_g)$$

$$\dot{K}_g = I_g$$

- Policy Implications: (i) No taxes should be levied on the income of the rich i.e. a tax holiday. The economy is allowed to grow before redistributive measures are introduced.
- (ii) All government savings beyond a level  $I_g$ , i.e. uncommitted government savings should be transferred to the poor in the form of capital grants. This alternative is preferable to that of expanding state capital.

### Regime II (Case B)

Sub period:  $t_1 \leq t \leq T$

Control Variables:  $\lambda = 1$ ,  $\mu = 1$

Co-State Variables:

$$q_1 = \frac{a}{\delta} \left[ 1 - e^{\delta(t-T)} \right]$$

$$q_4 = q_2 - q_3 = \frac{1}{\delta} \left[ \gamma_2 - (1-s_2)/\beta_2 + G \right] \left[ e^{\delta(t-T)} - 1 \right]$$

$$+ \frac{G e^{\delta(t-T)}}{s_2 / \beta_2} \left[ e^{-s_2 \beta_2 (t-T)} - 1 \right]$$

These equations represent the manner in which the shadow prices of capital accumulation should behave over the sub-plan period  $(t_1, T)$

State Variables: The capital accumulation equations are:

$$\dot{K}_1 = s_1 Z_1$$

$$\dot{K}_2 = s_2 \beta_2 K_2 + (s_g \gamma_3 K_g - I_g)$$

$$\dot{K}_g = I_g$$

Policy Implications: (i) Income of the rich in excess of the level  $Z$ , is transferred to the poor. That is the fruits of growth should be transferred to the poor during this phase.

(ii) As before all government savings beyond a level  $I_g$ , i.e. uncommitted government savings should be transferred to the poor in the form capital grants.

In the event of there being no switch from one regime to another during the plan period, the system would start in Case B and end in Case B. In such circumstances the results are the same as those shown in Regime II above.

## 7.5 An Extension

One of the simplifications made in the previous section was that of selecting the objective function to be maximised as consumption of the poor rather than the utility of consumption. This simplification led to a function which was linear in  $\lambda$  and  $\mu$  and hence the solution obtained was of the bang-bang type, with  $\lambda$  and  $\mu$  assuming extreme values of either 0 or 1. Particularly in the case of  $\lambda$ , the switch from 0 to 1, if a switch did take place, did seem too extreme a change.

An attempt will be made in this section to demonstrate how such extreme situations could be avoided in principle, with the use

of a suitable utility function. On account of the nature of the mathematical complexity involved, no attempt will be made to solve the problem completely. Instead, the emphasis will be on demonstrating how a value of  $\lambda$  lying between 0 and 1, i.e. an interior solution could be obtained. Such a value if obtainable, would tell us what the optimal proportion of income transfer (of income beyond  $Z_1$ ) ought to be.

We shall consider a utility function which is commonly used in the literature namely:

$$U(C) = \frac{1}{1-\nu} C^{1-\nu} \quad \dots (7.58)$$

Where  $\nu$  is a positive constant. As before  $C$  represents the consumption of the poor.

The marginal utility is given by:

$$U'(C) = C^{-\nu} \quad \dots (7.59)$$

Clearly  $U'(C) > 0$  and decreases as  $C$  increases, as one would expect. Thus the utility function defined by equation (7.58) is a concave one.

In this case the optimisation problem would be to

$$\text{Maximise} \quad \int_0^T \frac{1}{1-\nu} [\alpha_2 K_1 + (\alpha_1 K_1 - Z_1) + (1-s_2)\beta_2 K_2 + \gamma_2 K_2]^{1-\nu} e^{-\delta t} dt$$



subject to the constraints defined by equations (7.6)

Let the undiscounted Hamiltonian be defined by

$$H' = \frac{1}{1-\nu} \left[ \alpha_2 K_1 + (\alpha_1 K_1 - Z_1)\lambda + (1-s_2)\beta_2 K_2 + \gamma_2 K_g \right]^{1-\nu} \\ + q_1 s_1 \{ Z_1 + (1-\lambda)(\alpha_1 K_1 - Z_1) \} + q_2 \{ s_2 \beta_2 K_2 + (s_g \gamma_3 - I_g) \mu \} \\ + q_3 \{ I_g + (s_g \gamma_3 K_g - I_g)(1-\mu) \}$$

Then by applying Pontryagin's Maximum Principle as before the nature of the optimal solution could be explored.

In the case of an interior solution, the optimal value of  $\lambda$  is determined by the equation.

$$\frac{\partial H'}{\partial \lambda} = 0 \quad \dots (7.60) \\ \text{i.e. } \left[ \alpha_2 K_1 + (\alpha_1 K_1 - Z_1)\lambda + (1-s_2)\beta_2 K_2 + \gamma_2 K_g \right]^{-\nu} (\alpha_1 K_1 - Z_1) \\ = q_1 s_1 (\alpha_1 K_1 - Z_1) \dots (7.61)$$

As usual, this equation may be interpreted as:-

Marginal utility of consumption =  $q_1 \times (\text{Investment})$

Thus  $q_1$  is the price of capital accumulation of the rich measured in terms of the marginal utility of consumption of the poor.

From equation (7.61) we get

$$\alpha_2 K_1 + (\alpha_1 K_1 - Z_1)\lambda + (1-s_2)\beta_2 K_2 + \gamma_2 K_g = (q_1 s_1)^{-\frac{1}{\nu}} \dots (7.62)$$

$$\text{i.e. } \lambda = \frac{(q_1 s_1)^{-\frac{1}{\nu}} - [\alpha_2 K_1 + (1-s_2)\beta_2 K_2 + \gamma_2 K_g]}{(\alpha_1 K_1 - Z_1)} \dots (7.63)$$

This solution will be an interior solution so long as

$$\frac{(q_1 s_1)^{-\frac{1}{\sigma}} - [\alpha_2 K_1 + (1 - s_2)/\beta_2 K_2 + \gamma_2 K_g]}{(\alpha_1 K_1 - Z_1)} < 1 \dots (7.64)$$

$$\text{i.e. } \frac{(q_1 s_1)^{-\frac{1}{\sigma}} - \text{Earned Income of the poor}}{\text{Income of the rich}} < 1 \dots (7.65)$$

The above inequality would obviously depend upon the behavior pattern of  $q_1$  over the plan period and could be investigated through the canonical equation for  $q_1$ , which is

$$\dot{q}_1 - \delta q_1 = - \frac{\partial H'}{\partial K_1} \dots (7.66)$$

$$\text{i.e. } \dot{q}_1 - \delta q_1 = - \left[ \{ \alpha_2 K_1 + (\alpha_1 K_1 - Z_1) \lambda + (1 - s_2)/\beta_2 K_2 + \gamma_2 K_g \} \alpha_1 + q_1 s_1 \alpha_1 (1 - \lambda) \right] \dots (7.67)$$

From equations (7.61) and (7.67) we get

$$\begin{aligned} \dot{q}_1 - \delta q_1 &= - [q_1 s_1 \alpha_1 \lambda + q_1 s_1 \alpha_1 (1 - \lambda)] \\ &= - q_1 s_1 \alpha_1 \dots (7.68) \end{aligned}$$

$$\text{i.e. } \dot{q}_1 = (\delta - s_1 \alpha_1) q_1 \dots (7.69)$$

$$\text{Therefore } q_1 = A e^{(\delta - s_1 \alpha_1)t} \dots (7.70)$$

where A is an arbitrary constant which could be determined from boundary conditions.

On the basis of the terminal capital conditions specified in the previous section,  $q(T) = 0$  as before. It is clear from

inequality (7.65) that it cannot hold at  $t = T$ ; for, as  $t \rightarrow T$   $q_1(t) \rightarrow 0$  and  $(q_1)^{-\frac{1}{\gamma}} \rightarrow \infty$ . In these circumstances the solution (7.70) cannot hold in the neighbourhood of  $t = T$ . But supposing this solution is valid during the sub period  $t_1 \leq t \leq t_2$  within the interval  $(0, T)$  then the constant  $A$  could be found from the boundary conditions.

Thus at least in the case of the sub period  $(t_1, t_2)$  it would be possible in principle to find a value for the optimal proportion lying between 0 and 1.

The other significant departure arising from the more general utility function is that  $q_2$  and  $q_3$  are no longer independent of  $q_1$  as in the previous case. For the respective canonical equations are:

$$\begin{aligned} \dot{q}_2 - \delta q_2 = - & \left[ \{ \alpha_2 K_1 + (\alpha_1 K_1 - z_1) \lambda + (1 - s_2) \beta_2 K_2 + \gamma_2 K_g \} \right. \\ & \left. (1 - s_2) \beta_2 + q_2 s_2 \beta_2 \right]^{-\nu} \\ \dot{q}_3 - \delta q_3 = - & \left[ \{ \alpha_2 K_1 + (\alpha_1 K_1 - z_1) \lambda + (1 - s_2) \beta_2 K_2 \right. \\ & \left. + \gamma_2 K_g \} \gamma_2 + q_3 s_g \gamma_3 \right]^{-\nu} \dots (7.71) \end{aligned}$$

From equations (7.61) and (7.71) we get

$$\begin{aligned} \dot{q}_2 - \delta q_2 = - & [q_1 s_1 (1 - s_1) \beta_2 + q_2 s_2 \beta_2] \\ \dot{q}_3 - \delta q_3 = - & [q_1 s_1 \gamma_2 + q_3 s_g \gamma_3] \dots (7.72) \end{aligned}$$

It is clear from the above equations that the solutions for  $q_2$  and  $q_3$  would depend upon the solution for  $q_1$ .

## 7.6 Summary

An attempt was made in this chapter to demonstrate a further set of interrelationships that could exist between the processes of growth and redistribution of incomes, under conditions of optimality. The discussion was generally couched in the well known Tinbergen framework consisting of a model, an objective function, control variables, constraints and boundary conditions, but the techniques used were those of optimal control theory. At least in a rudimentary form it was possible to demonstrate how a control mechanism could be set up within a growth cum redistribution process and how two control variables i.e a tax amounting to a proportion  $\lambda$  of income of the rich beyond a tax limit  $Z_1$  and capital grants to the poor amounting to a proportion  $\mu$  of uncommitted government savings could be varied in order to maximise consumption of the poor over a plan period.

One of the important results that emerged from the analysis is that it is better to use uncommitted government savings for purposes of effecting capital grants to the poor for self employment rather than expand state capital. This agrees with the results of the previous chapters. The more important and perhaps the more interesting result is that the control variable  $\lambda$  should change from 0 to 1 when a "signal" emitted by the shadow price  $q_1$  indicates a change of regime. The policy

implication of this result is that incomes of the rich (who derive their income entirely from the modern sector) must be allowed to grow up to a point in an environment of tax holidays before redistributive measures are introduced. In these circumstances, income inequality could increase initially before a decline could be induced through redistributive measures. It is interesting to note that this behaviour pattern derived from considerations of optimality resembles the Kuznets curve discussed in the earlier chapters. Although it may not be an easy task to establish a definite link with adequate rigour, yet one could hypothesize that the Kuznets curve could be an outcome of a variety of forces including government policy in some way aimed at optimising the welfare of the poor over time.

One of the limitations in the solution obtained was that the parameter  $\lambda$  had to shift from one boundary value 0 to other 1 without proceeding through the intermediate values. That is the solution obtained was of the "bang-bang" type. As mentioned previously this was due to the fact that the objective function being linear in the control variables. However it was shown subsequently in section 7.5, how this somewhat unrealistic feature could be avoided by the use of a more general concave utility function. But the price that one has to pay is one of increased mathematical complexity. A complete solution in terms of such a utility function was not attempted.

CHAPTER 8A DISAGGREGATED APPROACH

"Each economic system - even that of an underdeveloped country - has a complicated internal structure. Its performance is determined by the mutual relations of its differentiated parts, just as the motion of the hands of a clock is governed by the gears inside" - Wassily Leontief (Input - Output Economics, page 41)

8.1 Introduction

Discussions carried out so far have been aggregative in character and consequently the policy implications derived have been of a broad and general nature. In this chapter, an attempt will be made to examine the implications of growth and redistributive objectives upon individual sectors of the economy.

Work in this area was pioneered by Narapalasingam {1970} and one of the questions he examined was how a change in income distribution might affect the productive structure of the economy. More recently, Pyatt and Roe {1977} have developed a comprehensive social accounting framework for Sri Lanka, and have inter alia demonstrated how it could be used as a convenient basis for development planning with special reference to employment and redistributive objectives. As an interesting follow up to their work, Pyatt and Round {1977} have examined the effect of sectoral expansion upon income distribution and have ranked sectors in terms of relative

efficiency in improving the distribution of incomes as well as in terms of a utility index based on alternative weighting schemes. This work has been extended in another direction by Roe and Tyler {1977a} who identified key sectors of the Sri Lankan economy in relation to alternative social objectives. Roe and Tyler {1977b} have also demonstrated how a Social Accounting Matrix (SAM) could provide a clearer insight into the relationships between productive activities and income distribution, employment and regional imbalances and how SAM could be used as an effective basis for welfare planning in Sri Lanka.

One of the aspects not covered in this area of investigation concerns the optimal allocation of resources over individual sectors of the economy in relation to a given social objective and the ranking of sectors according to their relative efficiencies in the use of resources. Rankings so obtained need not necessarily be the same as those obtained in the above mentioned studies. For, if a unit expansion in output of a sector A is more favourable than that of a sector B in relation to a given social objective, yet this ordering could get reversed if the capital-output ratio of A is sufficiently high in relation to B. That is, from the standpoint of resource allocation B may be more favourable than A, in attaining a given social objective.

The central theme of this chapter will be that of inquiring into optimal patterns of resource allocation in relation to alternative social objectives. The resource considered here is investible capital in the hands of government. The main alternative social objectives are the growth objective represented by the increase of Gross National Product, the redistributive objective represented by the increase of income of the poor and the employment objective, all of which are represented respectively as functions of gross output of individual sectors. Other social objectives considered are those of increasing the income of the urban poor, rural poor and the estate poor. We shall examine the manner in which each of these social objectives could be maximised subject to a budget constraint, a foreign exchange constraint as well as output, market and interdependency constraints, and study in particular the implications of growth and redistributive objectives.

The question of resource allocation over the sectors of the economy is of special significance due to two more reasons. Firstly, in the formulation of national plans one of the first problems we come across is that of deriving broad investment magnitudes for individual sectors. Secondly, the establishment of sectoral priorities as well as investment targets becomes a pre-requisite for the evaluation of individual projects for inclusion in national plans.



The main features of the model used in this chapter, are outlined in the section below:

## 8.2 The Model

The starting point of our analysis is the semi-disaggregated 12 sector SAM used by the Roe and Tyler {1977b}. It is possible to carry out the same analysis on the basis of a fully disaggregated 48 sector model of the Sri Lankan economy. But the 12 sector model was preferred firstly because the optimisation problem could be kept at a less cumbersome form and secondly the 12 sectors in semi-disaggregated form roughly corresponds to the major sectors for which investment targets are set out in the formulation of national plans in Sri Lanka. The twelve sectors are:

1. Tea
2. Rubber
3. Coconut
4. Paddy (Rice growing)
5. Other Agriculture
6. Agricultural Processing (Includes rice milling)
7. Mining
8. Traditional Industry
9. Modern Industry
10. Construction
11. Trade and Transport
12. Services.

The gross output of each of these sectors will be denoted by  $X_1, X_2, \dots, X_{12}$  respectively. In respect of each sectoral output  $X_i$ , we shall regard the inputs from other sectors to be a constant proportion of  $X_i$ . Likewise, the value added by each sector, the income shares of rich and poor and employment will be regarded as being directly proportional to gross output of that sector.

The assumption of fixed coefficients is no doubt a debatable one. For, it could be argued that the relationships between gross output and intermediate inputs or between gross output and income shares of the poor could be non-linear in character. But in the absence of empirical evidence regarding the exact nature of the production functions, the fixed coefficient assumption seems inevitable. Therefore, the results obtained must necessarily be regarded as first approximations. Nevertheless such results could give us useful insights into the nature of interdependencies in the economy. Moreover these results could be made use of to derive guidelines for action in terms of the orders of magnitude involved.

The other implicit assumption in our analysis is the invariance of input-output coefficients over time. Although there are methods such as the RAS technique developed along with the Cambridge Growth Model {1963} , we need not use this

technique since the time period considered in our optimisation problem is relatively short. In fact the optimisation problem considered here only deals with annual allocations of investment and of foreign exchange.

Let  $x_1, x_2, \dots, x_n$  denote annual increments in the gross outputs  $X_1, X_2, \dots, X_n$ . Since the question of resource allocation is related to annual budgets, we shall formulate the objective function in terms of the  $x$  s and write it as

$$W = \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n \quad \dots (8.1)$$

Where the  $\alpha$  s are constants characterizing the objective function discussed. For example, for the growth objective i.e. that of maximising national income, the  $\alpha$  s will represent the value added per unit of gross output in each sector; for the redistributive objective the  $\alpha$  s will represent the increase in income of the poor per unit of gross output. Likewise, in the case of the employment objective the  $\alpha$  s will represent employment generated per unit of gross output in each sector.

The budget constraint and the foreign exchange constraint may be specified as

$$v_1 x_1 + v_2 x_2 + \dots + v_n x_n \leq B \quad \dots (8.2)$$

$$\text{and } e_1 x_1 + e_2 x_2 + \dots + e_n x_n \leq E \quad \dots (8.3)$$

where  $v_1, v_2, \dots, v_n$  are the capital to gross-output ratios,  
 $e_1, e_2, \dots, e_n$  are the foreign exchange components of gross output



where  $\underline{f}$  represents the vector of increases in final demand. Therefore inequalities (8.6) amounts to the statement that

$$(I-A)\underline{x} = \underline{f} \quad (8.9)$$

Two other sets of constraints have to be introduced in order to make the model more realistic namely, supply constraints and market constraints. Supply constraints represent the physical upper limits to which sectoral outputs could be expanded during a period of one year. For example in the case of a crop, the availability of land or in the case of an industrial project the gestation period could impose an upper limit. Market constraints are more obvious; in the case of a good which cannot be internationally traded (e.g. construction), the domestic demand would impose an upper limit to output. But for sectors producing internationally traded goods, or international sectors in the terminology of Tinbergen {1966}, (e.g. Tea, Rubber, Coconut), market constraints are less obvious. Nevertheless there are limits to which exports of one commodity could be expanded in any one year since it takes time to enter into new markets, particularly when they are highly competitive, and it would appear reasonable to assume that market constraints exist even in the case of international sectors.

Supply constraints and market constraints may be represented by the equations

$$x_i \leq \bar{x}_i \quad \dots \quad (8.10)$$

$$\text{and} \quad f_i \leq \bar{f}_i \quad (8.11)$$

$$\text{or } (I - A)\tilde{x} \leq \tilde{f} \quad \dots (8.12)$$

where  $\bar{x}_1, \bar{f}_1$  represent the respective upper limits.

Thus the optimisation problem consists of maximising the objective function  $W$  subject to the constraints specified above. For purposes of greater clarity, the optimisation problem in its entirety may be recapitulated as follows:

$$\text{Maximise } W = \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n \quad (8.1) \text{ (Objective function)}$$

Subject to:

$$v_1 x_1 + v_2 x_2 + \dots + v_n x_n \leq B \quad (8.2) \text{ (Budget Constraint)}$$

$$e_1 x_1 + e_2 x_2 + \dots + e_n x_n \leq E \quad (8.3) \text{ (Foreign Exchange Constraints)}$$

$$x_i \geq 0, \quad i = 1 \dots 12 \quad (8.4) \text{ (Non-negativity Constraints)}$$

$$\left. \begin{aligned} (1-a_{11})x_1 - a_{12}x_2 - \dots - a_{1n}x_n &\geq 0 \\ -a_{21}x_1 + (1-a_{22})x_2 - \dots - a_{2n}x_n &\geq 0 \\ \dots &\dots \dots \\ -a_{n1}x_1 - a_{n2}x_2 - \dots + (1-a_{nn})x_n &\geq 0 \end{aligned} \right\} \begin{array}{l} (8.5) \text{ (Interdependency} \\ \text{Constraints)} \\ n = 12 \end{array}$$

$$x_i \leq \bar{x}_i, \quad i = 1, \dots, 12 \quad (8.10) \text{ (Supply Constraints)}$$

$$\left. \begin{aligned} (1-a_{11})x_1 - a_{12}x_2 - \dots - a_{1n}x_n &\leq \bar{f}_1 \\ -a_{21}x_1 - (1-a_{22})x_2 - \dots - a_{2n}x_n &\leq \bar{f}_2 \\ \dots &\dots \dots \\ -a_{n1}x_1 - a_{n2}x_2 - \dots + (1-a_{nn})x_n &\leq \bar{f}_n \end{aligned} \right\} \begin{array}{l} n = 12 \end{array}$$

Thus, the optimisation problem consists of maximising the objective function  $W$  subject to 50 constraints in all.

Having obtained the optimal solution the optimal allocation of resources could be easily derived as follows:

Let  $x_i^*$  represent the optimal solution. Then since  $v_i$  represents the corresponding capital to gross output ratios, the optimal allocation of capital over the sectors are represented by  $v_1 x_1^*, v_2 x_2^* \dots v_n x_n^*$ .

In matrix form the optimal capital allocation is represented by:

$$\begin{bmatrix} v_1 & & & 0 \\ & v_2 & & \\ & & \ddots & \\ 0 & & & v_n \end{bmatrix} \begin{bmatrix} x_1^* & & & 0 \\ & x_2^* & & \\ & & \ddots & \\ 0 & & & x_n^* \end{bmatrix}$$

or  $\hat{\underline{v}} \hat{\underline{x}}^*$  ..... (8.13)

where  $\hat{\underline{v}}, \hat{\underline{x}}^*$  represent the diagonalised forms of the vectors  $\underline{v}$  and  $\underline{x}^*$

If the capital budget is the binding constraint, then

$$\sum_{i=1}^n v_i x_i^* = B \quad \text{i.e.} \quad \underline{v}' \underline{x}^* = B \quad \dots (8.14)$$

Likewise the optimal allocation of foreign exchange is represented by

$$\hat{\underline{e}} \hat{\underline{x}}^* = 1 \quad \dots (8.15)$$

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<sup>1</sup> If the foreign exchange constraint is binding (both constraints could be binding), then  $\underline{e}' \underline{x}^* = E$

Since the problem formulated above has more than one objective function it is also possible to examine the implications of a given optimal solution (arising from one objective), upon another objective. For example, suppose  $\alpha_1^1, \alpha_2^1, \dots, \alpha_n^1$  are the coefficients of the growth objective, and let  $x_1^*, x_2^* \dots x_n^*$  be the optimal solution. Then if  $\alpha_1^2, \alpha_2^2, \dots, \alpha_n^2$  denote the coefficients of the redistributive objective function ( $\alpha_1^2$  represents the income of the poor from sector 1) then the increase of income of poor =  $\alpha_1^2 x_1^* \dots$  (8.16)

Likewise, if  $\alpha_1^3, \alpha_2^3, \dots, \alpha_n^3$  represent the coefficients of the employment objective ( $\alpha_1^3$  represent the employment generated per unit of gross output in sector 1) then increase in employment =  $\alpha_1^3 x_1^* \dots$  (8.17)

Conversely, if we take the redistributive objective as given, the optimal solution so obtained could be made use of to find out the implications upon the growth objective. Thus, the optimal solutions so obtained would open up yet another area of inquiry in regard to the interrelationships between redistributive and growth objectives in terms of resource allocation and these could be reduced to numerical form.

The next section sets out the data framework used.

### 8.3 The Data Framework

Values of all parameters characterising the objective functions



were derived from the 12 sector SAM presented in Roe and Tyler {1977}. Six objective functions were developed. They are:

- Obj. 1. The Growth Objective - where  $\alpha_1$  represents value added in sector i as a proportion of gross output of sector i.
- Obj. 2. The Redistributive Objective - where  $\alpha_1$  represents the income of the poor earned from sector i as a proportion of gross output of sector i.
- Obj. 3. The Employment Objective - where  $\alpha_1$  represents the employment generated in thousands per Rs.1 m expansion of gross output of sector i.
- Obj. 4. The Urban Welfare Objective - where  $\alpha_1$  represents the income of the urban poor earned from sector i, as a proportion of gross output of sector i.
- Obj. 5. The Rural Welfare Objective - where  $\alpha_1$  represents the income of the rural poor earned from sector i as a proportion of gross output of sector i.
- Obj. 6. The Estate Welfare Objective - where  $\alpha_1$  represents the income of the estate poor earned from sector i as a proportion of gross output of sector i.

Values of the parameters of the respective objective functions are shown in the Table 8.1.

Table 8.1

Objective Functions

Objec- tive	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$x_{11}$	$x_{12}$
Obj.1	0.679	0.856	0.917	0.862	0.853	0.125	0.780	0.324	0.429	0.552	0.816	0.726
Obj.2	0.584	0.679	0.522	0.666	0.500	0.069	0.385	0.212	0.131	0.243	0.289	0.302
Obj.3	0.703	0.511	0.090	0.635	0.156	0.039	0.193	0.198	0.051	0.066	0.156	0.139
Obj.4	0.002	0.005	0.031	0.011	0.045	0.009	0.018	0.045	0.032	0.050	0.084	0.073
Obj.5	0.078	0.406	0.419	0.656	0.446	0.053	0.367	0.166	0.099	0.187	0.199	0.206
Obj.6	0.505	0.267	0.071	-	0.010	0.006	-	0.001	0.001	0.006	0.006	0.023

In order to calculate the capital to gross output ratios  $v_1$ , estimates of the respective capital output ratios were first obtained from Tyler {1976}. These estimates were adjusted by multiplication by a constant factor so as to obtain a weighted (by value added in each sector) capital-output ratio of 3.41, which is the estimate used in Chapter 4. This procedure was carried out since the sectoral capital-output ratios appeared to be understated. These ratios were then multiplied by the respective value added to gross output ratios (derived from the SAM) in order to arrive at capital to gross output ratios required for the formulation of the budget constraint. The foreign exchange components of gross output  $e_1$  were directly estimated from the 12 sector SAM for purposes of formulating the foreign exchange constraint. The table below shows the coefficients of the budget and foreign exchange constraints.

Table 8.2

Coefficients of the Budget and Foreign Exchange Constraints

Coefficient	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$x_{11}$	$x_{12}$
$v_i$	1.690	2.173	2.349	0.463	0.947	0.205	3.730	1.106	2.156	0.925	2.540	8.255
$e_i$	0.087	0.032	0.017	0.026	0.029	0.099	0.028	0.106	0.145	0.038	0.025	0.078

The total budget B was taken as Rs.1000 million so as to be broadly representative of capital formation by Government and Public Corporations<sup>1</sup> during 1970. The upper limit (E) on extra foreign exchange needed for meeting additional foreign exchange requirements for increases in gross output was taken as Rs.100 million.

The input-output coefficients were also derived from the 12 sector SAM and the "interdependency" constraints were formulated in terms of equation (8.9). The coefficient of these constraints are shown in Table 8.3.

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<sup>1</sup>The exact figure for 1970 was Rs.1020.9 million.

Source: Central Bank Report, 1974, Appendix III, Table 7.

Table 8.3

Coefficients of the Interdependency Constraints

SEC-TOR	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$x_{11}$	$x_{12}$
1	1.000	-	-	-	-	-	-	-	-	-	-	-0.0012
2	-	1.0000	-	-	-	-	-	-0.0059	-	-	-	-
3	-	-	1.0000	-	-	-0.1223	-	-	-	-	-	-0.0036
4	-	-	-	0.9382	-	-0.5012	-	-	-0.0013	-	-	-
5	-0.0127	-0.0027	-	-	0.9485	0.0154	-0.0183	-0.0461	-0.0013	-0.0026	-	-0.0237
6	-	-	-	-	-0.0060	0.9832	-	-0.1389	-0.0006	-	-	-0.0243
7	-	-	-	-	-	-	1.0000	-0.0015	-0.0019	-0.0388	-	-0.0006
8	-0.0174	-	-	-	-0.0103	-0.0094	-	0.9287	-0.0188	-0.0530	-0.0161	-0.0030
9	-0.0949	-0.0642	-0.0156	-0.0309	-0.0271	-0.0119	-0.0827	-0.0267	0.7866	-0.0714	-0.0430	-0.0188
10	-	-	-	-	-	-	-0.0092	-	-	0.9160	-0.0034	-0.0285
11	-0.0637	-0.0294	-0.0156	-0.0071	-0.0125	-0.0575	-0.0550	-0.0840	-0.0895	-0.1160	0.9536	-0.0376
12	-0.0069	-0.0053	-0.0+04	-0.0079	-0.0005	- 0.0020	-	-	-0.0006	-0.0031	-0.0057	0.9915

In the case of supply constraints, an attempt was made to take account of maximum annual increases in output that have taken place during the past ten years under the best of possible circumstances. Working papers prepared for the Medium Term Investment Programme 1978-82 were also consulted so as to ensure that estimates of physical upper limits on increases in output were far as possible in agreement with these estimates. Market constraints were formulated on the basis of what would seem possible for the market in each sector to absorb. Special attention was paid to Sectors 10, 11 and 12 which are dominated by goods which cannot be internationally traded. In these cases, constraints were fixed on the basis of maximum increases in demand likely to be experienced. The estimates obtained are set out in the Table 8.4 .

Table 8.4

Supply and Market Constraints

Sector	Percentage increase assumed in fixing supply constraints	Supply Constraints (Rs.m) $\bar{x}_i$	Market Constraints (Rs.m) $\bar{f}_i$
1. Tea	3%	26	172
2. Rubber	2%	8	110
3. Coconut	7%	40	65
4. Paddy	10%	113	49
5. Other Agriculture	15%	270	799
6. Agric. Processing	50%	1010	873
7. Mining	20%	22	28
8. Trad. Industry	20%	269	1014
9. Modern Industry	10%	154	648
10. Construction	20%	382	169
11. Trade & Transport	15%	447	208
12. Services	10%	165	158

The somewhat low market constraint for Paddy should not be interpreted as an undue restriction. As could be seen from the 12 sector SAM, the output of paddy first enters the Agricultural Processing sector for milling before release to the market.

#### 8.4 Results

The optimal solutions<sup>1</sup> in respect of growth and redistributive objectives are set out in Tables 8.5 and 8.6 below. The optimal allocations of investment as well as the implications of these solutions in regard to growth, income distribution, employment and additional foreign exchange requirements, derived from equations (8.13), (8.15), (8.16) etc. are also set out in these tables.

The solutions obtained indicate that whether the objective is one of growth or of increasing the incomes of the poor, the main thrust of development ought to be in agriculture and that the investment in agriculture should be about 50 per cent of total investment. This is not altogether surprising since agriculture is considered to be the leading sector in the Sri Lankan economy from the point of view of potential for development. Value added

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<sup>1</sup> Optimal solutions were obtained by the use of the MINILP package developed by the Computer Department of the University of Warwick.



per unit of gross output is greatest in the agricultural sector i.e. potential for growth is greatest in agriculture. Further the majority of the poor live in rural and estate areas (see Chapter 2), primarily engaged in agricultural activity. The two solutions are remarkably similar; the values in respect of the first six variables are identical. In fact, the values for the first five sectors namely Tea, Rubber, Coconut, Paddy and Agriculture are the upper limits fixed by supply constraints (see Table 8.4). This means that optimality with respect to the growth objective or with respect to the redistributive objective calls for the development of these five sectors up to levels set by physical constraints. The potential indicated for Sector 5, namely Other Agriculture (which includes subsidiary food crops, livestock and minor export crops) is particularly significant. Output in this sector has steadily increased during the last few years.<sup>†</sup>

The identical values obtained in respect of the first six variables in the two solutions suggests that agricultural development provides a strong linkage between growth and redistributive objectives. These two objectives appear to be completely complementary as far as agricultural development is concerned. These results also strengthen the arguments in Chapter 3 <sup>that</sup> growth in the agricultural sector could have been a

<sup>†</sup> Output of subsidiary food crops and of minor export crops have doubled during the period 1970-75 (Source: Central Bank of Ceylon).

major cause for reduction of income inequality in Sri Lanka in recent years.

Table 8.5  
Variant I - The Growth Objective  
Optimal Solution

Sector	Increase in Gross output (Rs.m.)	Investment Allocation Invest- ment (Rs.m.)	Percen- tage	Foreign Exchange Require- ment (Rs.m.)	Increase in GDP (Rs.m.)	Increase in Income of the poor (Rs.m.)	Increase in Emplc ment (Thousand)
1. Tea	26.00	43.94	4.39	2.26	17.65	15.18	18.28
2. Rubber	8.00	17.38	1.74	0.26	6.85	5.43	4.09
3. Coconut	40.00	93.96	9.40	0.68	36.68	20.88	3.60
4. Paddy	113.00	52.31	5.23	2.94	97.41	75.26	71.76
5. Other Agric.	270.00	255.70	25.57	7.83	230.31	135.00	42.12
6. Agric. Process.	211.41	43.34	4.33	20.93	26.43	14.58	8.24
7. Mining	7.28	27.15	2.72	0.20	5.68	2.80	1.41
8. Trad. Industry	18.13	20.05	2.01	1.92	5.87	3.84	3.59
9. Modern Industry	43.13	92.98	9.30	6.25	18.50	5.65	2.20
10. Construc- tion	184.90	171.03	17.10	7.03	102.06	44.93	12.20
11. Trade & Transport	61.74	156.82	15.68	1.54	50.38	17.84	9.63
12. Services	3.07	25.34	2.53	0.24	2.23	0.93	0.43
Total	988.66	1000.00	100.00	52.08	600.05	342.32	177.55

Table 8.6

Variant II - The Redistributive Objective  
Optimal Solution

Sector	Increase in Gross output (Rs.m)	Investment Allocation Invest- ment (Rs.m)	Percen- tage	Foreign Exchange Require- ment (Rs.m)	Increase in GDP (Rs.m)	Increase in Income of the poor (Rs.m)	Increase in Employ- ment (Thousands)
1. Tea	26.00	43.94	4.39	2.26	17.65	15.18	18.28
2. Rubber	8.00	17.38	1.74	0.26	6.85	5.43	4.09
3. Coconut	40.00	93.96	9.40	0.68	35.68	20.88	3.60
4. Paddy	113.00	52.31	5.23	2.94	97.41	75.26	71.76
5. Other Agric	270.00	255.70	25.57	7.83	230.31	135.00	42.12
6. Agric. Process.	211.41	43.34	4.33	20.93	26.43	14.58	8.24
7. Mining	7.32	27.30	2.73	0.20	5.71	2.82	1.41
8. Trad. Industry	43.30	48.00	4.80	4.60	14.06	9.20	8.59
9. Modern Ind.	43.39	93.55	9.36	6.29	18.62	5.68	2.21
10. Construc- tion	184.85	171.00	17.10	7.02	102.04	44.92	12.20
11. Trade & Trans Transport	50.69	128.75	12.87	1.27	41.36	14.65	7.91
12. Services	3.00	24.77	2.48	0.23	2.18	0.91	0.42
Total	1001.06	1000.00	100.00	54.51	599.30	344.51	180.83

As regards variables 7-12, representing the non agricultural sectors, optimal values under Variant I differ from those obtained under Variant II. The differences are slight except in the case of traditional industry and trade and transport. The redistributive objective calls for greater emphasis in traditional industry

indicating the scope of this sector in raising the income levels of the poor. Trade and transport generates income primarily for the upper income groups<sup>1</sup> and as one would expect a reduction is indicated under Variant II.

In overall terms, the above results show a slight trade-off between growth and redistributive objectives. Under Variant I, the increase in GDP is Rs.600.05 million and the increase in income of the poor is Rs.342.32 million. Whereas under Variant II, the increase in GDP is Rs.599.30 million but the increase in income of the poor is Rs.344.51 million. Although this trade-off is hardly significant, probably a more appreciable trade-off would result if the optimisation problem is carried out recursively over time. This exercise is not being attempted on account of the extremely complicated nature of the problem when formulated in multisectoral terms. In terms of employment, Variant II shows a slight advantage over Variant I, but foreign exchange requirements are slightly higher. In both variants, the major contribution to employment are shown from the Paddy sector and from Other Agriculture.

Table 8.7 presents the optimal solution in respect of the employment objective (Variant III), and related results. Except in the case of coconut, no change is indicated in the variables

<sup>1</sup> As could be seen from Objective 1, the proportion of value added to gross output is 0.816, but income of the poor as a proportion of gross output is only 0.289 (see Objective 2)

relating to the subsectors of agriculture. This suggests that as far as agricultural development is concerned, there is a great degree of complementarity between the employment objective and the growth objective, and in turn with the redistributive objective. In other words, agricultural development provides a strong positive linkage between the growth, redistributive and employment objectives.

The decline in the value of the variable representing coconut is not altogether surprising. As, could be seen from Objective 3 (see Table 8.1) the generation of employment per unit output of coconut is much lower than those of tea, rubber and paddy, although the coefficients are of comparable magnitude in the case of Objectives 1 and 2. Arising from the lower value for the variable representing coconuts, a slightly lower investment in agriculture is shown in Variant III. Investment in agriculture as a proportion of total investment is of the order of 47 per cent as compared with a proportion of about 50 percent in Variants I and II.

Table 8.7Variant III - Employment ObjectiveOptimal Solution

Sector	Increase in Gross output (Rs.M)	Investment <u>Allocation</u> Invest- ment (Rs.M)	Percen- tage	Foreign Exchange Require- ment (Rs.M)	Increase in GDP (Rs.M)	Increase in Income of the Poor (Rs.M)	Increase in Employ- ment (Ths.)
1. Tea	26.00	43.94	4.39	2.26	17.65	15.18	18.28
2. Rubber	8.00	17.38	1.74	0.26	6.85	5.43	4.09
3. Coconut	25.87	60.77	6.08	0.44	23.72	13.50	2.33
4. Paddy	113.00	52.31	5.23	2.93	97.40	75.26	71.76
5. Other Agriculture	270.00	255.70	25.57	7.83	230.31	135.00	42.12
6. Agriculture Process	211.44	43.35	4.33	20.93	26.43	14.59	8.25
7. Mining	0.93	3.47	0.35	0.03	0.73	0.36	0.18
8. Trad. Industry	269.00	297.52	29.75	28.51	87.16	57.03	53.26
9. Modern Industry	34.22	73.78	7.38	4.96	14.68	4.48	1.75
10. Construc- tion	11.79	10.91	1.09	0.45	6.51	2.86	0.78
11. Trade & Transport	48.02	121.97	12.20	1.20	39.18	13.88	7.49
12. Services	2.29	18.90	1.89	0.18	1.66	0.69	0.32
Total	1020.56	1000.00	100.00	69.98	522.28	338.26	210.61

Perhaps the most significant feature in the results at Table 8.7 is the marked variation in the values of the Variables 7-12 representing the non-agricultural sectors. The optimal solution under Variant III shows a significant increase in respect of traditional industry, highlighting

the potential this sector offers in regard to employment creation - a somewhat obvious result since the implicit capital labour ratio is as low as Rs.5,585 as compared with Rs.42,263 for modern industry.<sup>1</sup> The optimal value for the sector is in fact the upper limit fixed by conditions of supply (see Table 8.4)

Variant III also indicates that construction per se is not that labour intensive and that the emphasis on this sector ought to be very much less from the standpoint of employment creation. Decreases of varying magnitudes are also indicated in respect of the sectors of Mining, Modern Industry, Trade and Transport, and Services.

In overall terms, an interesting trade-off is seen between the growth and employment objectives. Under Variant I the increase in GDP is Rs.600.05 million and the increase in employment is 177,550. Whereas under Variant III, the increase in GDP is Rs.552.28 million but the increase in employment is 210,610. However, the general result that emerges is that under conditions of optimality, the growth, redistributive and employment objectives are to a greater degree complementary than competitive. Results relating to Variant IV i.e. the optimisation of the urban welfare Function (the share of income of the urban poor) are presented in Table 8.8.

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<sup>1</sup> The capital-labour ratio for modern industry ranges between Rs.25,000 and 145,000 (see Central Bank of Ceylon, Review of the Economy, 1975. p.60)

Table 8.8

Variant IV - Urban Welfare Objective

Optimal Solution

Sector	Increase in Gross output	Investment Allocation		Foreign Exchange Require- ment	Increase in GDP	Increase in Income of poor	Increase in Income of Urban poor	Increase in Employ- ment
	(Rs.M)	(Rs.M)	Invest- ment Percent- age	(Rs.M)	(Rs.M)	(Rs.M)	(Rs.M)	(Thousand)
1. Tea	-	-	-	-	-	-	-	-
2. Rubber	1.48	3.22	0.32	0.05	1.27	1.00	0.01	0.76
3. Coconut	4.55	10.69	1.07	0.08	4.17	2.37	0.14	0.41
4. Paddy	19.90	9.21	0.92	0.52	17.15	13.25	0.22	12.64
5. Other Agric.	270.00	255.70	25.57	7.83	230.31	135.00	12.15	42.12
6. Agric. Process.	37.15	7.62	0.76	3.68	4.64	2.56	0.33	1.45
7. Mining	7.62	28.42	2.84	0.21	5.94	2.93	0.14	1.47
8. Trad. Industry	250.91	277.52	27.75	26.60	81.29	53.19	11.29	49.68
9. Modern Ind.	39.98	86.20	8.62	5.80	17.15	5.24	1.28	2.04
10. Construction	184.82	170.96	17.10	7.02	102.02	44.91	9.24	12.20
11. Trade & Transport	54.88	139.40	13.94	1.37	44.78	15.86	4.61	8.56
12. Services	1.34	11.06	1.11	0.10	0.97	0.40	0.10	0.19
Total	872.63	1000.00	100.00	53.26	509.69	276.71	39.51	131.52



As one would expect the optimal solution indicates a shift away from the major agricultural crops i.e. Tea, Rubber, Coconut and Paddy. But the emphasis on other agriculture remains unchanged at its upper limit. This is because subsidiary food crops could be grown in urban areas (which are much less urbanized by Western Standards), and particularly in large extents of land in semi-urban areas. As could be seen from Objective Function 4, (see Table 8.1) income of the poor per unit of gross output in Other Agriculture i.e. the coefficient of  $x_5$  is the fourth highest, compared with rest of the coefficients in this objective function.

The other sectors highlighted in the optimal solution are Traditional Industry, Construction and Trade and Transport. An investment allocation of nearly 59 percent of total investment is indicated for these three sectors alone.

The optimal solution also shows that preoccupation with urban welfare could lead to lesser increases <sup>in</sup> GDP and employment. However, no meaningful comparisons could be made with variants I, II and III since we are dealing with only a very small segment of the community.

Results relating to Variants V and VI are presented in Tables 8.9 and 8.10

Table 8.9

Variant V - Rural Welfare Objective  
Optimal Solution

Sector	Increase in Gross output	Investment Allocation		Foreign Exchange Require- ment	Increase in GDP	Increase in income of poor	Increase in Income of Urban poor	Increase in Employment
	(Rs.m.)	Invest- ment (Rs.M)	Percen- tage	(Rs.M.)	(Rs.M)	(RS.M.)	(Rs.m.)	(Thousands)
1. Tea								
2. Rubber	8.00	17.38	1.74	0.26	6.85	5.43	3.25	4.09
3. Coconut	40.00	93.96	9.40	0.68	36.68	20.88	16.76	3.60
4. Paddy	113.00	52.31	5.23	2.94	97.41	75.26	74.13	71.76
5. Other Agric.	270.00	255.70	25.57	7.83	230.31	135.00	120.42	42.12
6. Agric. Process.	211.41	43.34	4.33	20.93	26.43	14.58	11.20	8.24
7. Mining	7.38	27.53	2.75	0.21	5.76	2.84	2.71	1.42
8. Trad. Industry	83.69	92.56	9.26	8.87	27.12	17.74	13.89	16.57
9. Modern Ind.	41.71	89.93	8.99	6.05	17.89	5.46	4.13	2.13
10. Construction	184.85	170.99	17.10	7.02	102.04	44.92	34.57	12.20
11. Trade & Transport	52.34	132.94	13.29	1.31	42.71	15.13	10.42	8.17
12. Services	2.83	23.36	2.34	0.22	2.05	0.85	0.58	0.39
Total	1015.21	1000.00	100.00	56.32	595.25	338.09	292.06	170.69

Table 8.10

Variant VI - Estate Welfare ObjectiveOptimal Solution

Sector	Increase in Gross output (Rs.M)	Investment Allocation		Foreign Exchange Require- ment (Rs.M)	Increase in GDP (Rs.M)	Increase in Income of poor (Rs.M)	Increase in Income of Estate poor (Rs.M)	Increase in Employ- ment (Thousands)
		Invest- ment (Rs.M)	Percen- tage					
1. Tea	26.00	43.94	4.39	2.26	17.65	15.18	13.13	18.28
2. Rubber	8.00	17.38	1.74	0.26	6.85	5.43	0.21	4.09
3. Coconut	40.00	93.96	9.40	0.68	36.68	20.88	2.84	3.60
4. Paddy	113.00	52.31	5.23	2.94	97.41	75.26	-	71.76
5. Other Agric.	270.00	255.70	25.57	7.83	230.31	135.00	2.70	42.12
6. Agric. Process	211.41	43.34	4.33	20.93	26.43	14.58	1.27	8.24
7. Mining	7.29	27.19	2.72	0.20	5.69	2.81	-	1.41
8. Trad. Industry	17.90	19.80	1.98	1.90	5.80	3.79	0.02	3.54
9. Modern Ind.	42.52	91.67	9.17	6.17	18.24	5.57	0.04	2.17
10. Construction	184.98	171.10	17.11	7.03	102.11	44.95	1.11	12.21
11. Trade & Transport	48.54	123.28	12.33	1.21	39.61	14.03	0.29	7.57
12. Services	7.31	60.33	6.03	0.57	5.31	2.21	0.17	1.02
Total	976.95	1000.00	100.00	51.98	592.09	339.69	21.78	176.00

Since both variants represent non-urban situations, the similarity of the two solutions are not altogether surprising. However, there are a few obvious, but important differences. Tea has a zero value in Variant V, indicating the absence of any impact of Tea upon the rural community. Again, Traditional industry receives low emphasis under Variant VI. The contribution to GDP and employment, under both variants are substantially higher than in the Variant V and comes very close to those of Variants I, II and III. This shows that preoccupation with development issues of the non-urban sector has a closer correspondence to national development objectives than those of the urban sector - a result which is to be expected since the vast majority of the poor live in non urban areas (see Chapter 2). However, both solutions have some elements in common with Variant IV. The closest are the values for other agriculture and construction, which highlight the potential of these sectors in raising the incomes of the poor whether they be in urban rural or estate areas.

### 8.5 Summary

In contrast to the approaches in the previous chapters which were essentially aggregative in character, the approach adopted in this chapter was to consider growth and redistribution in terms of individual sectors of the economy. An attempt was made to inquire into the interrelationships that could exist between growth and redistributive objectives in terms of individual

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sectors of the economy, under conditions of optimality. The technique of optimisation over time, of the type appearing in the previous chapter was not attempted here in view of somewhat intractable mathematical difficulties that could arise.<sup>1</sup> The method adopted here was one of one-period optimisation and the entire problem was formulated in terms of a simple linear programming model, the constraints imposed being the capital budget, foreign exchange, supply, market and interdependency constraints. Optimal allocations of capital resources over the major sectors of the economy were obtained with respect to the growth, redistributive employment objectives as well as urban, rural and estatewelfare objectives. It was thus possible to compare the respective optimal allocations with a view to establishing possible links between the objectives considered, via individual sectors of the economy.

The first important result that emerged from the analysis was that whether the objective be one of growth or of raising the income of the poor or of employment creation, the main thrust of development ought to be in agriculture and that, subject to the validity of the assumptions made, particularly those regarding the constraints, the investment in agriculture should be around half of total investment. Conversely, agricultural

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<sup>1</sup> A rigorous discussion would have perhaps required a major extension of the Chakravarty model {1965} or the Von Neumann Model {1945}.

development emerges as a strong link between the growth, redistributive and employment objectives. These results lend further support to the great emphasis on agriculture placed by successive governments in Sri Lanka.

The second interesting set of results obtained were the significant differences in the allocation of resources over the non agricultural sectors with respect to the objective functions considered. For example, in comparison with the growth objective, the redistributive objective called for a greater emphasis on traditional industry and a lesser emphasis on trade and transport. Likewise the employment objective called for a much greater emphasis <sup>on</sup> traditional industry and a much lesser emphasis on construction.

The other interesting results obtained were the trade-offs between the respective objective functions. The trade-off between the growth and redistributive objectives appeared to be much less than between the growth and employment objectives. However, in general the growth, redistributive and employment objectives showed a greater degree of complementarity than competitiveness under conditions of optimality.

objectives  
Variants IV to VI concerning the urban, rural and estate welfare/  
also yielded some interesting results. For example, although the

urban welfare objective called for a shift away from the main crops, the emphasis on other agriculture remaining unchanged; the potential for agricultural diversification was thus highlighted. Results also indicated that preoccupation with urban welfare could lead to lesser increases in GDP and employment as compared with the other alternatives. Results also emphasised the converse, namely that preoccupation with development issues in the non urban sector had a closer correspondence to national development objectives.

Some of the above results have, no doubt, only confirmed the intuitively obvious. Perhaps the more important contribution of the preceding analysis is that it has provided a framework for resource allocation. The respective optimal solutions show the manner in which resources ought to be allocated ideally, so that the actual process of resource allocation has a framework to work with, although it is still at best a first approximation. Furthermore, some idea of the broad allocation of resources as between sectors is an essential pre-requisite for the evaluation of individual projects prior to inclusion in national plans. The preceding analysis has also demonstrated the implications of one optimal solution upon other objectives as well as on the use of foreign exchange.

CHAPTER 9POLICY IMPLICATIONS

"A good choice of policy depends on some subtle aspects of the economic environment and social preferences" -  
R.M. Solow (Growth Theory, An Exposition, p.91)

### 9.1 Formulation of Economic Policy

The urgencies of seeking solutions to certain economic problems, particularly in a developing country, are such that often policies are formulated on an ad hoc basis. Apart from the question of urgency, ad hoc methods of policy formulation are some times inevitable due to the general paucity of information about the economic environment. Although several examples could be cited to show that economic policies formulated on such methods have not been altogether wrong, yet in general these methods could have adverse effects upon some part of the economy or on the economy as a whole in the long run. For, the economy is just not a collection of separate entities but a system with a number of interdependencies amongst its components, as for example between output, employment, wage rates, shares of income, government expenditure, money supply, rates of interest and other variables. Therefore a remedy in one regime of the economy may result in a problem in another, which may not be obvious at the time of decision making. It follows therefore that a specific problem cannot be looked upon as a problem by itself but a problem in a system. In other words we have to look at the



system as a whole and adopt a systems approach to problem solving.

It is in this context that a model of the economy could be of great help. For, it enables us to set out in a relatively concise form the interdependencies between the important components. As noted earlier, the aim of model building is not to obtain an exact representation of reality but a representation of the more important facets of reality as a system of interdependent elements. Having constructed a model, it is possible for one to study the impact of the changes in one element upon the other elements of the system and deduce policy implications.

Some amount of caution is however necessary in the use of models. For, a model can be no more accurate than the set of assumptions upon which it is built. Further, even if we have built a fairly realistic model, much of its usefulness will depend upon the quality of data fed into the model. We have already discussed the limitations of the model developed in this study and of the data used and it is important to keep these limitations in mind when attempting to deduce policy implications. In fact the results may be somewhat different if some values of the parameters differ from those estimated.

In view of the nature of limitations of the model and inadequacies in the quality of data available, what would seem more prudent is to obtain information about types of effect which certain policies if formulated will produce rather than to devise precise quantities or targets for the key variables. However, this does not altogether preclude us from deriving broad magnitudes wherever possible, for purposes of guidance.

In this chapter we shall attempt to bring together only the policy conclusions derived at various stages of this study. General conclusions and some of the important analytical results will be recapitulated in Chapter 10.

## 9.2 Capital for Self Employment

Subject to the assumption that the estimates of the parameters used in this study are reasonably accurate, considerations of the static form of the model demonstrated that policies which stimulate increases of capital for self employment are preferable to those that expand state capital, from the standpoint of improving the incomes of the poor. Dynamic considerations of the model amplified this result further by showing that the rate of growth of incomes of the poor depends to a great extent upon the output - capital ratio relating to the capital owned by the poor and their marginal savings rate. The

parameter  $\beta_2$  which is the product of these two variables assumed a crucial role in the numerical simulations carried out in Chapter 5; slight changes in  $\beta_2$ , produced dramatic changes in the incomes of the poor at a terminal date. Further, our discussions in Chapter 7 concerning the question of optimising the incomes of the poor over time revealed that uncommitted investible resources in the hands of the government should preferably be converted into capital grants to the poor rather than using these resources for expanding state capital.

Thus the broad strategy indicated consists of:

- (a) expanding capital for self employment rather than expanding state capital;
- (b) raising the output of capital for self employment; and
- (c) raising the marginal savings rate of the poor.

In tangible terms, this points out to the desirability of implementing projects such as land settlement schemes (where farmers are allocated state land for purposes of cultivation), greater efforts in raising productivity of capital which in the case of land would take the form of providing irrigation facilities, more distribution of fertilizer, the intensification of agricultural extension services, and to the encouragement of savings and investment amongst the lower income groups.

As noted in Chapter 3, successive governments in Sri Lanka have implemented land settlement schemes since the enactment of the Land Development Ordinance in 1935. Nearly 75,000 families had been settled over the period 1946-71.<sup>1</sup> Likewise efforts have been made to raise agricultural productivity through a number of measures such as agricultural research, extension services, distribution of fertilizer and irrigation schemes. It is significant to note<sup>that</sup> the largest ever irrigation cum power project in the country namely the Mahaweli project has just been initiated. It envisages the settlement of 140,000 families over a land area of approximately 900,000 acres.<sup>2</sup>

### 9.3 Subsidies

One of the interesting results that emerged from the numerical simulations in Chapter 5 was that significant increases in the rate of growth of subsidies could lead only to a marginal increase in the income of the poor at a terminal date. For, when the growth rate of subsidies increases by fivefold from 1 per cent to 5 per cent the terminal income of the poor increases only from 103.27 to 105.51. subject to the assumption that the estimates of the various parameters are reasonably accurate. The underlying reason is the diminution in the growth of government income which in turn would lead to a

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<sup>1</sup> Source: Administration Reports of the Land Commissioner 1970/71

<sup>2</sup> Government of Sri Lanka, Medium Term Investment Programme 1979-83.

diminution in the growth of wage income of the poor. These results highlighted the desirability of moving away from a system of consumer subsidies.

As noted in Chapter 3 the present climate of thought has moved distinctly away from the preoccupation with consumer subsidies. The removal of the rice subsidy from the richer half of the population could be seen to be a major step in this direction; subsidies are now restricted to a target group consisting of household earning less than Rs.300 per month. A further movement in the direction of reducing subsidies would appear to be desirable. For, in terms of static considerations of the model (see Chapter 5) we noted how the partial conversion of a consumer subsidy into a capital grant would produce more desirable results in terms of consumption after a time lag of about 3 years. However, one of the limitations of the alternative of effecting capital grants to the poor is that only certain segments of the poor are benefited, whereas a subsidy such as the rice subsidy is spread widely amongst the poor, While there does not seem to be a clear cut answer to this problem, a partial solution might be to spread as far as possible, at least in a geographical sense, land development projects or rural industrial projects.

#### 9.4 Redistribution of Consumption

The investigations in Chapter 4 revealed that while income

inequality would decline with the introduction of specific redistributive measures, total income too would decline, the underlying reason being the diminution in total savings due to a differential savings rate as between the rich and the poor. Thus income redistribution and the growth of total income emerged as competing objectives. We then examined an alternative strategy of redistribution of consumption. That is, the strategy whereby the amount obtained from the rich as taxes is not transferred entirely to the poor, but only a proportion  $c_1/c_2$  of this amount, where  $c_1$  and  $c_2$  are the marginal rates of consumption of the rich and poor respectively. Under this scheme, the balance is to be retained by government for direct investment. As noted in Chapter 4 this introduced in formal terms a system of redistribution where there was no loss of savings and a special assumption about government according to which the amount retained by government was saved and invested. The outcome of this strategy was noted to be a decline in income inequality while total income increased. Thus, under a scheme of consumption redistribution, the redistributive and growth objectives emerged as complementary objectives, subject to the assumption that the "Pleasure" effect did not take place.

In practice the implementation of such a scheme would take the shape of a more judicious use of government revenues, so that

the balance left over after the transfer is saved and invested. A broader issue which we were unable to capture within the framework of the model concerns the production of consumer goods. A programme of consumption redistribution may also necessitate the stepping up of supplies of goods consumed by the poor (eg. specific categories of wage goods) so as to enable them to purchase these goods from the increased incomes they would receive from such transfers.

Thus the main policy conclusion we derived was that a system of consumption redistribution as defined in this study is preferable to one of income redistribution, from the point of view of pursuing the twin objectives of growth and redistribution of incomes.

#### 9.5 The Role of the Modern (Private) Sector

One of the interesting analytical results obtained in Chapter 4 was that  $\frac{\partial Y_2}{\partial \alpha_1} > 0$  where  $\alpha_1$  was identified as the main parameter associated with the growth of incomes of the rich in a situation where the rich derive the major part of their income from the modern (private) sector rather than from the government. It was clear from the definition of  $\alpha_1$  that it is the parameter which represents the growth rate of the modern (private) sector. Also in Chapter 6 we discussed the circumstances under which

the modern (private) sector must be allowed to grow before redistributive measures are introduced, from the standpoint of optimising the incomes of the poor.

Subject to the assumption that the estimates of the parameters used are reasonably accurate the policy implication of these results is that a slowing down of the growth of the incomes of the rich could adversely affect the incomes of the poor in the long run, the underlying reason being the linkage between the two income groups through the wage income the poor receive from employment in the modern sector. It was apparent therefore that if the incomes of the poor are to be bettered through employment, expansion of the modern (private) sector should receive high priority.

It is significant to note that a number of meaningful steps in this direction have been already taken. Perhaps the most important of these is the establishment of the Greater Colombo Economic Commission (GCEC), to be in charge of an area of approximately 180 sq. miles in the northern outskirts of Colombo city. Major export oriented industries both local and foreign are to be attracted into Investment Promotion Zones within the GCEC through a number of incentives such as tax holidays averaging to 5 years, duty free imports of raw materials and machinery, exemption from tax for foreign personnel employed in



the industrial projects and nominal tax rates after the expiry of the tax holiday period. A wide range of infrastructure facilities are also to be provided. However it should be noted that the extent of foreign ownership lessens the linkages between the socio-economic groups implicit in the model. The role of private foreign investment has not been covered in our study.

A number of concessions have also been offered to private industry outside the GCEC. These include tax holidays to small and medium scale industry, removal of licence control on 85 per cent of imported raw materials, and a more liberal administrative framework for the approval of new industries. All these will have the effect of increasing industrial output in the modern (private) sector and thereby create greater employment opportunities. There is scope to take further steps in this direction such as for example the provision of greater credit facilities to industrialists, stepping up of extension services and the systematic removal of undue delays in the system of approval of new industries.

#### 9.6 The Role of Foreign Aid

Another interesting analytical result we obtained from Chapter 5 was that foreign aid will contribute more to the rate of growth of incomes of the poor than that of the rich, subject to the

assumption that the estimates of the parameter in our study are reasonably accurate. The underlying reason for this is that foreign aid is largely directed towards the government projects which primarily provide incomes for the poor. The policy implication of this result is that the seeking of foreign aid is potentially in conformity with a growth cum redistribution strategy.

A number of interesting developments in this area have been witnessed in the recent past. The volume of aid pledged at the fourteenth Sri Lanka Aid Group meeting held in May 1978 is the highest on record since the inception of the Aid Consortium. The total pledge amounts to approximately US \$ 380 million of which US \$ 129 million constitutes an outright grant. These figures exclude the foreign assistance under negotiation for the Mahaweli Project referred to earlier.<sup>1</sup>

## 9.7 Population Policies

In Chapter 4 we also set out to examine the impact of differential population growth rates upon the distribution of income. The results obtained illustrated somewhat dramatically the inadequacy of redistribution measures unless they are accompanied by measures to curb the growth rate of the poor. The analysis demonstrated that it is only when the rate of growth of population of the poor

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<sup>1</sup> See Budget Speech of November 1978

could be reduced to a level below that of the rich, that there is some scope to reduce income inequality in per capita terms. Such a task is not impossible. For, we noted that fertility is continuing to decline at a rate fast enough to halve fertility in 10 years. The implicit policy conclusion is that population control policies should be primarily aimed at the lower income groups. This could be facilitated to some extent if conscious efforts are made to locate family planning units in geographical areas where the poor are predominant. Likewise educational and publicity programmes could be directed primarily at the poor households.

#### 9.8 The Incentive Effect

In Chapter 6 we attempted to take account of skilled manpower as factor of production in the production functions used in the model. The first important analytical result obtained was that taxation to a point which would affect the supply of skilled manpower, due to the leisure-income preference relationship, could adversely affect the incomes of the poor. In other words it is possible that income redistribution could be self defeating particularly when it involves taxation of skilled personnel. Although we were not able to solve the problem completely due to difficulties in obtaining estimates of some of the parameters, it was also possible to demonstrate how the entire growth process could slow down due to a decrease in the

supply of skilled manpower.

The implicit policy conclusion is that possibilities of incentive losses should be carefully taken into account when tax proposals affecting professional incomes are formulated. One approach in this direction is to review the present structure of "earned income" reliefs and revise taxes downwards as appropriate. But the Budget for 1979 (see Budget Speech of November 1978) has already made a far reaching change, namely to exempt all government employees from income tax in respect of employment income obtained from government. This was primarily aimed at giving relief to the professional categories employed in government. Although it may not be necessary to extend the same facility to the private sector, in view of the generally higher system of rewards prevailing in the private sector, yet it may be opportune to consider the question of granting greater reliefs on professional incomes earned.

## 9.9 Allocation of Resources

An attempt was made in Chapter 8 to inquire into the interrelationships that could exist between growth and redistributive objectives in terms of individual sectors of the economy under conditions of optimality. The objective functions considered included the growth objective, the redistributive objective, the employment objective as well as the urban, rural and estate welfare objectives. The problem was formulated in terms of a simple linear programming model, subject to a budget, foreign exchange, supply,

market, and interdependency constraints. We were thus able to obtain an optimum allocation of resources over the major sectors of the economy in respect of each objective function.

The first important policy conclusion that emerged from the analysis was that whether the objective be one of growth or raising the incomes of the poor or of employment creation, the main thrust of development should be in agriculture and that the investment in agriculture development emerged as a strong investment. Thus agricultural development emerged as a strong linkage between the growth, redistributive and employment objectives. It is interesting to note that the actual allocation of resources contemplated by government in regard to public sector projects over the period 1979 to 1983 comes fairly close to this result. In fact the proposed capital expenditure on agriculture (inclusive of the Mahaweli Project) represents approximately 42 per cent of the total investment proposed. Thus there appears to be further scope to move towards an optimal allocation by diverting more resources to agriculture.

Another major policy conclusion we derived from this analysis concerned the allocation of resources over non-agricultural sectors. The growth and redistributive objectives called for a greater emphasis on traditional industry and a lesser emphasis on trade and transport. The analysis also indicated the employ-

ment potential of traditional industry. This indicates the high priority that ought to be assigned towards the development of small and medium scale industries as well as cottage industries. In tangible terms this calls for greater support facilities such as establishment of common services centres to cater to the needs of the small industrialists, greater extension services, more credit facilities and training personnel in government who would be concerned in the development of the small scale industry.

The results of Chapter 8 also provided a framework within which questions of resource allocations could be examined. For example the analysis showed that the growth objective called for an allocation of about 50 per cent from the total resources available for investment, those for modern industry, construction, trade and transport being respectively 9.30, 17.10 and 15.68 per cent. The broad allocation of resources between sectors thus obtained, although at best is a first approximation could serve as a useful starting point when the actual process of resource allocation has to be considered.

CHAPTER 10CONCLUDING REMARKS

"Economic theory, in its purest and most abstract form, can be treated as a system of logic, having no more immediate ethical contact than a proposition in Euclidean geometry ... Yet no scientific investigation however abstract or detached, can entirely escape the probability of having ethical consequences, remote though this possibility may at first appear" - W.S. Vickrey (Goals of Economic Life).

We began this study by inquiring into the nature and extent of income inequality in Sri Lanka. The main conclusion was that significant progress had been made in moving towards a more equitable distribution of incomes in Sri Lanka over the period 1963-73. The share of income of the richest decile of households declined sharply from 40.60 per cent in 1963 to 28.03 per cent in 1973, whilst the share of the poorest decile rose significantly from 1.50 per cent in 1963 to 2.79 per cent in 1973; significant improvements had taken place in the other lower deciles too.

We also noted that the overall reduction of income inequality had come about due to the reduction of income inequality in the urban and rural sectors; there had been no reduction of

income inequality in the estate sector. But in real terms, non-urban incomes per spending unit had witnessed steady rates of growth in contrast to a decline in the case of urban spending units, indicating that conditions had moved in favour of the non urban population over a period 1963-73. Also there were no marked disparities in consumption as between urban, rural and estate spending units in 1973, except in the case of a few items namely the relatively higher level of consumption of protein foods in the urban sector, inadequate housing facilities in the estate sector, low expenditure on education by estate spending units, and the relatively higher expenditure on durable consumer goods by urban spending units. It was also noted that there had been a growth and redistribution of incomes over the period 1963-73. For, a little less than a third of the total increase in real income during this period had reached the bottom forty per cent of the spending units whose share in 1963 was nearly 15 per cent of total income. Thereby, their share of income rose to over 19 per cent by 1973.

An attempt was made in Chapter 3 to isolate the main factors that could have contributed towards the reduction of income inequality in recent years. Although there was no clear cut way of quantifying the contributions from each factor, it was noted that reduction



of income inequality would have come about partly from welfare measures such as the subsidies on food, health, education and transport, as well as minimum wage policies and partly from agricultural development primarily aimed at domestic production of essential food items, which had the effect of increasing farmer incomes.

As stated at the outset, the main purpose of this study was to explore the interrelationships that could exist between the processes of growth and redistribution of incomes by means of a formal model representative in Sri Lanka. The scenario for this study was set, so to speak, by the Chenery-Ahluwalia model for distribution with growth {1974} . Their study attempted to introduce somewhat concisely a unified theory of redistribution with growth, bringing out clearly some aspects of interdependent growth amongst the various socio-economic groups through employment linkages. We extended this model firstly by incorporating the government as a separate entity participating in a growth cum redistribution process and <sup>secondly</sup> by introducing a number of other features such as the role of financial institutions, direct and indirect taxes, subsidies, foreign aid, incentive losses and also considerations of optimal growth.

Although the assumptions made at the various stages of development of the model were somewhat simple, the analytical results obtained

seemed to be more complex than one would have expected. This indicates the nature of complexities that could exist in reality within a process of interdependent growth. One of the important analytical results which we were able to derive was that showing how the model could generate the Kuznets pattern in general. A further generalization incorporating specific redistributive measures enabled us to obtain, as a limiting case, a situation where the Kuznets pattern was indiscernible. We hypothesised that this limiting case should represent the Sri Lankan situation where the trend in the recent past has been one of a steady decline of income inequality through specific redistributive measures.

The analytical results demonstrated the importance of raising the productivity of capital owned by the poor and of raising their marginal savings rate, the possible adverse effects upon the income of the poor by slowing down the growth of the modern (private) sector and the urgency of directing the population control policies towards the poor if income inequality is to be reduced in per capita terms. We also showed how the redistributive and growth objectives could become competing objectives under a scheme of direct income redistribution and in contrast how these two objectives could be made complementary under a scheme of consumption redistribution as defined in this study. The model developed was extended in Chapter 5 by the incorporation of

financial institutions, direct and indirect taxes, and foreign aid. With the incorporation of these elements, the analytical solutions obtained assumed further complexity and demonstrated the somewhat complicated nature of the interactions amongst the various elements of the model. In particular the analytical solutions demonstrated the possible impact of subsidies, higher productivity of capital and foreign aid upon the incomes of the poor within a growth cum redistribution process, and set out a framework for evaluating the policy options available. For example, the results highlighted that increasing the growth rate of subsidies could have only a limited impact upon the income of the poor in the long run whereas, a marginal increase in the productivity of capital owned by the poor could bring about substantial benefits.

The role of skilled manpower in the development process was given explicit recognition in Chapter 6, by incorporating it as a factor of production. We were thus able to demonstrate at least in a rudimentary form the possible adverse effects on economic growth arising from a lowering of the supply of skilled manpower due to the interaction of taxes upon the leisure-income preference relationship. That is, with increased taxes, there would be a tendency for skilled personnel to trade-off leisure for work or even emigrate, resulting in a downturn of development activity.

As noted at the outset, the question of optimality within a growth cum redistribution process has not been adequately covered in the literature; the closest was the contribution by Hamada {1967}. But as noted earlier he had considered only the limited case where workers do not save at all. Further, only one policy instrument was considered. What we attempted to do in Chapter 7 was to extend the approach by Hamada firstly by allowing a positive savings rate by the poor, secondly by introducing two policy instruments, thirdly by assigning a specific role to government. More generally we sought to introduce considerations of optimality into a Chenery-Ahluwalia type of model, and explore the inter-relationships that could exist between the processes of growth and redistribution of incomes under conditions of optimality.

The entire discussion was couched in the well known Tinbergen framework consisting of a model, an objective function, control variables, constraints and boundary conditions. The optimisation problem was formulated in terms of Hamiltonians and analytical solutions were obtained using Pontryagin's Maximum principle. We were thus able to demonstrate how a control mechanism could be set up within a growth cum redistribution process and how two control variables i.e. a tax on the income of the rich and capital grants to the poor could be varied in order to maximise welfare of the poor over a finite time horizon, subject to given constraints.

The solutions also showed how the shadow prices of three different types of capital behaved over a finite time horizon. In particular, the shadow price of capital in modern (private) sector presented an interesting possibility of how it could signal a change of regime - the first regime where such capital is allowed to expand in a tax free environment and a second regime where surpluses are transferred to the poor. We also noted that in such circumstances, income inequality could increase initially before redistributive forces induce a decline. Thereby, we attempted to derive a theoretical result which would correspond to the Kuznets pattern. The manner in which the "bang bang" type of solution obtained could be avoided by the use of a more general concave utility function was also discussed.

A major departure from the aggregative character of the discussions was made in Chapter 8 by trying to inquire what the growth, redistributive and employment objectives mean in terms of individual sectors of the economy. Thereby we sought to capture a further set of interrelationships that might exist between the objective functions considered. Unlike in the case of Chapter 7 where the problem was conceived of as one of optimising an objective function over time, here the problem was formulated in terms of a simple linear programming model, subject to a budget, foreign exchange, supply, market and interdependency constraints,

the time period involved being one year. The analysis showed that agricultural development provides a strong linkage between the growth, redistributive and employment objectives and that investment in agriculture should be around half the total investment. In other words these objectives assumed complementarity with respect to agricultural development. The analysis also demonstrated some trade-offs in resource allocation over the non-agricultural sectors with respect to the objective functions considered. For example, the redistributive objective called for greater emphasis on traditional industry and lesser emphasis on trade and transport, whereas the emphasis was somewhat reversed under the growth objective. More importantly, the analysis provided a broad framework for resource allocation between sectors although it was still at best a first approximation.

The policy conclusions derived from the analytical results were recapitulated in Chapter 9. But as noted therein a number of these policies are already being implemented. However, it is not immediately obvious whether the policies which are being implemented are mutually consistent. Perhaps the main justification for the model building exercise carried out in this study lies in the fact that we were able to derive these policy conclusions somewhat rigorously, from a model which has attempted to work towards a unified theory of redistribution with growth, subject to the assumption that estimates of the parameters used were reasonably accurate.

There are several directions in which this study could be extended. Firstly, the number of income groups could be increased so as to make it more representative of the real situation. But since analytical results are bound to be intractable when this extension is introduced computerisation would seem inevitable. The second direction in which this study could be further extended is by examining the question of resource allocation over the sectors over time; we had considered only a one period optimisation. This may be extended for a time horizon of say 5 years to begin with. But a rigorous exercise may necessitate a major extension of the multisector optimisation model of Chakravarty 1965 and is bound to present numerous mathematical difficulties. More importantly, the main task ahead would be one of trying to derive improved estimates of the main parameters and thereby develop a firmer empirical foundation to start with. For, the final results obtained could be more accurate than the assumptions and data used.

In conclusion, it is hoped that this study has contributed, at least in a small way to the current efforts towards developing a unified theory of redistribution with growth. It is also our earnest hope that this study will stimulate further discussion and research on this subject, which is of considerable relevance today.

APPENDIX I  
(To Chapter 2)

TABLE A I - 1

Sectoral Composition of Gross National Product  
at Constant (1959) Factor Cost Prices - 1977

	<u>Rs. Million</u>	<u>Percentage</u>
1. Agriculture, Forestry, Hunting and Fishing	3,828	32.0
2. Mining and Quarrying	311	2.6
3. Manufacturing	1,505	12.6
4. Construction	480	4.0
5. Electricity, Gas, Water and Sanitary Services	37	0.3
6. Transport, Storage and Communication	1,198	10.0
7. Wholesale and Retail Trade	1,623	13.6
8. Banking, Insurance and Real Estate	229	1.9
9. Ownership of Dwellings	360	3.0
10. Public Administration and Defence	703	5.9
11. Services	1,703	14.3
12. Gross Domestic Product	11,977	-
13. Net Factor Income from Abroad	- 25	-0.2
14. Gross National Product	11,952	100.00

Source: Central Bank of Ceylon, Annual Report  
1977, Table 2, p.7.



APPENDIX II  
(To Chapter 2)

TABLE A II - 1

Composition of Exports and Imports 1975-77

(a) Exports 1975-77 (at Current Prices)

	<u>1975</u>	<u>1976</u>	<u>1977</u>
	Rs. Million		
1. Tea	1,932	2,100	3,503
2. Rubber	654	890	931
3. Coconut Products	397	382	335
4. Minor Agricultural Crops (Selected Items)	171	231	338
5. Gems (Precious and Semi Precious Stones)	180	261	298
6. Industrial Exports	542	782	866
7. Other Exports	48	156	345
8. Re-exports	10	14	23
	<u>3,934</u>	<u>4,816</u>	<u>6,639</u>

(b) Imports 1975-77 (at Current Prices)

	<u>1975</u>	<u>1976</u>	<u>1977</u>
	Rs. Million		
1. Food and Drink	2,520	1,491	2,181
2. Textiles and Clothing	20	49	150
3. Other Consumer Goods	111	149	203
4. Intermediate Goods	1,888	2,259	2,648
of which (i) Fertilizer	(208)	(99)	(51)
(ii) Petroleum	(872)	(1,164)	(1,441)
5. Investment Goods	653	641	746
6. Unclassified Imports	<u>59</u>	<u>54</u>	<u>79</u>
	<u>5,251</u>	<u>4,643</u>	<u>6,007</u>

Source: Central Bank of Ceylon, Annual Report 1977,  
Tables 10 and 11, pp. 24-25.

APPENDIX III  
(To Chapter 2)

A note on the Consumption Patterns  
in Urban, Rural and Estate Sectors  
and the Changes in the Distribution  
of Income

The purpose of this note is two fold. Firstly, we shall compare the consumption/expenditure patterns in regard to food, clothing housing and other selected items as between urban, rural and estate sectors. Secondly, we shall examine how the distribution of income within each sector changed over time.

The main sources of information used were the Consumer Finance Survey of 1973 and the Housing Census of 1971 carried out by the Department of Census and Statistics. Other sources such as the Central Bank Annual Reports and the "Population of Sri Lanka" (1974) by the Department of Census and Statistics were also used in the latter part of this note.

The CFS 1973 sets out the average quantities of food items consumed per head in terms of a cross classification of 20 items of food (with sub items) by 9 income groups. However, for the purpose of this comparative account, 18 main items were selected after summarising the sub items and we have considered only the overall averages with a view to presenting the main features of the disparities between urban, rural and estate consumption patterns. The results are presented in Table A III - 1 below

TABLE A III - 1

Average Quantities of Food Consumed Per Head for Two Months in 1973					
	<u>Unit</u>	<u>Urban</u>	<u>Rural</u>	<u>Estate</u>	<u>All Island</u>
1. Rice	lb.	29.50	32.36	32.92	31.88
2. Wheat Flour	lb.	3.86	4.66	24.16	6.40
3. Bread	lb.	10.19	5.88	2.23	6.34
4. Other grain	lb.	1.86	3.32	1.68	2.88
5. Starch Food (Includes Yams)	lb.	1.76	2.79	1.88	2.50
Sub Total 1-5	lb.	47.17	49.01	62.87	50.00
6. Vegetables	oz.	109.14	136.12	120.93	129.57
7. Pulses	oz.	18.54	18.74	33.58	20.35
8. Meat	oz.	21.80	7.30	7.09	10.01
9. Fish	oz.	72.29	46.23	16.78	48.27
10. Eggs	No.	4.14	1.87	2.80	2.93
11. Milk	Bottle	2.42	1.25	4.19	1.76
12. Tinned Milk	oz.	8.04	3.41	1.52	4.10
13. Milk Products	oz.	0.67	0.12	0.06	0.22
14. Sugar	oz.	71.64	19.36	56.80	65.18
15. Jaggery	oz.	1.68	0.81	2.39	1.84
16. Fruits	No.	1.88	0.97	1.88	1.23
17. Fruits (Tinned/ Dried)	oz.	0.04	45.29	-	32.12
18. Coconuts	No.	12.49	14.14	13.09	13.73

Source: CFS 1973, Tables S 589-592, pp. 494-501.

The above table indicates that in 1973, rural and estate households were able to enjoy higher levels of per capita consumption in cereals and starch foods (items 1 to 5), vegetables and pulses than urban households. But in the case of meat, fish and eggs, the rural and estate households were worse off, and so was the case in regard to

tinned milk and milk products. Estate households were however, better off in regard to milk consumption, and this could be explained in terms of the predominance of dairy farming in the estate areas. There appears to have been a wide disparity between urban and rural sugar consumption, even when one takes into account the consumption of jaggery, a sugar substitute. A part of the explanation for this disparity could be an underestimate of jaggery consumption. Moreover, this disparity may have also been partly offset by the high per capita dried fruit consumption in rural areas. Thus, it would appear from this table that except in the case of meat, fish, eggs and milk, there did not seem to be marked disparities in consumption per capita, as between urban, rural and estate households. This disparity was perhaps partly offset by the higher consumption of vegetables and fruit in rural areas and high consumption of protein containing pulses in the case of the estate areas.

In the case of clothing, the average expenditure per spending unit could be used to assess disparities since there is no marked variation in the price of cloth as it is distributed throughout the country through consumer co-operatives at more or less uniform prices. The average expenditure on clothing and footwear incurred by urban, rural and estate

spending units in 1973 are reported to be as follows:

TABLE A III - 2

Average Expenditure per Spending Unit per month  
on Clothing and Footwear

Item	Urban Rs.	Rural Rs.	Estate Rs.	All Island Rs.
1. Clothing	28.10	19.44	27.49	21.91
2. Footwear	2.90	1.36	0.92	1.61
Total	31.00	20.80	28.41	23.52

Source: CFS 1973, Table S 593 - 596

It is clear from the above table that rural spending units spend much less than their urban counterparts on clothing and footwear. One cannot immediately interpret this result as a rural spending unit being worse off than an urban spending unit as regards clothing and footwear. For, the rural life style and rural occupations such as paddy cultivation call for much less clothing. On the other hand, estate spending units with an income level comparable to that of a rural spending unit spent almost as much as an urban spending unit on clothing and footwear. This could be explained in terms of the necessity for greater expenditure on clothing by estate households as they live in estate areas mainly located in the hill country regions where the climate is relatively cold.

We shall now examine the position regarding the next important item, namely housing. As in the case of food, expenditure cannot be taken to be a criterion. For, urban rents are very much higher than rural rents. It is therefore necessary to use physical measures for purposes of comparison. The table below attempts this comparison.

TABLE A III - 3

Floor Space per person and Number of Persons  
per Room - 1971

Sector	Floor Space per person (1)	No. of Persons per Room (2)	Percentage of housing units below minimum Standard (3)
Urban	Sq.ft. 86.64	2.34	45.9
Rural	79.21	2.19	39.0
Estate	51.93	3.38	46.9
All Island	81.85	2.65	41.2

Source: Department of Census and Statistics,  
Housing Census - 1971.

Note 1. Room include bed rooms, living rooms, sitting rooms and dining rooms.

Note 2. Computations for columns (1) and (2) were made from Tables 8 and 6 respectively.

Note 3. Column (3) was obtained from Table 9 and is intended to measure the extent of over crowding. The extent of overcrowding had been calculated for Table 9 on the basis of instances where:-

- (i) No. of occupants exceeded 2 in Housing Units of less than 100 sq. ft.
- (ii) No. of occupants exceeded 4 in Housing Units of 100 - 250 sq. ft.

- (iii) No. of occupants exceeded 6 in Housing Units of 250 - 500 sq. ft.
- (iv) No. of occupants exceeded 8 in Housing Units of 500 - 1000 sq. ft.

The above table indicates that although floor space per person in the rural sector is less than that in urban areas, there is less overcrowding in rural areas than in urban areas. The other significant feature is the unsatisfactory nature of housing conditions in estate areas, indicated by the higher number of persons per room and the higher percentage of sub standard housing.

The indicators presented in Table A III - 3 above gives us essentially a quantitative picture of disparities in housing standards. This must necessarily be supplemented with a qualitative picture in order to appreciate the real nature of the disparities in housing standards Table A III - 4 below attempts to do this.

TABLE A III - 4  
Housing Units classified by Structural  
Type and basic amenities - 1971

Sector	Percentage of Housing Units of				Percentage of Housing Units with piped water on tap	Percentage of Housing Units with Elec. su
	Permanent type	Semi-Permanent type	Temporary type	Total		
Urban	62.1	28.1	9.0	100.0	45.3	34.5
Rural	30.8	61.5	7.7	100.0	4.8	2.8
Estate	12.1	85.1	2.8	100.0	74.8	4.1
All Island	34.7	57.9	7.4	100.0	20.1	9.0

Source: Department of Census and Statistics, - Housing Census 1971. Tables 3, 10, 13A.

Note: Percentages of the respective structural types were derived from Table 3. These types were defined as follows:

- Permanent - where walls, floor and roof are all made of durable products like brick, cement, tile, asbestos sheets etc.
- Temporary - where walls are made of cadjan, palmyrah or other inferior and non-durable material.
- Semi-permanent - where a mixture of both durable and non-durable materials have been used.

On the basis of conventional urban housing standards and amenities, it would appear from the above table that urban households are better off on the whole. On the other hand, if we regard some of the semi-permanent structures (i.e where a mixture of both durable and non durable materials are used) as acceptable, results of Table A III - 4 may not be so obvious. An environmentalist could further argue that village wells are a better source of pure water than large scale water supply systems. Whilst it is true that some sections of the urban community, particularly the higher income groups enjoy comfortable housing conditions, the general urban scene today is one of high density, overcrowding, acute shortage of rental accommodation, and sub standard housing. In contrast, these problems are not prevalent in the rural areas. As much as 76.9 per cent of the housing units are owner occupied in contrast to an extent of 47.7 per cent in urban areas.



A brief comment also seems necessary in regard to the rest of the items in the basket of goods. These items are:

- (i) Medical Services
- (ii) Other goods and services, which include
  - education,
  - personal spending,
  - betel,
  - tobacco,
  - alcoholic beverages,
  - recreation and entertainment,
  - transport and communication,
  - servants,
  - ceremonial,
  - litigation,
  - laundry,
  - gifts,
  - fuel and light.
- (iii) Durable consumer goods, which consists of jewellery and other items.

According to the CFS 1973 (Tables S 594 - 596), significant differences do exist only in the case of three items, namely education, transport and communication and "other" durable consumer items. In the case of the first item, the monthly expenditure was as low as to 1.58 per estate spending unit as compared with Rs.10.32 in the case of an urban spending unit and Rs.6.22 in the case of a rural spending unit. In the case of the second item, the urban expenditure per spending unit was nearly 3 times that of an estate spending unit and over 50 per cent that of a rural spending unit. Differences were even more marked in the case of "other" durable consumer goods. Urban expenditure per spending unit was over 3 times that of an estate spending unit and over 5 times that of a rural spending unit. This disparity is also clearly seen in terms

of the ownership of durable consumer goods given in CFS 1973 Table 40. For instance cookers (Kerosene and other) are owned by 33.4 per cent of urban households in contrast to 5.7 per cent in the case of rural households and 4.1 in the case of estate households. Again, the percentages of households owning refrigerators in urban, rural, estate areas are respectively 4.9, 0.5 and 0.8. The respective percentages in the case of telephones are 1.0, 0.1 and 0.6.

Thus the general picture that seems to emerge from this comparative account is that except in the case of a selected number of items in a common basket of goods, the disparities are not so marked as between urban, rural and estate households. The first exception is the consumption of meat, fish, eggs and milk, in regard to which the urban households were better off. Inadequacy of housing facilities in estate areas were the second element of disparity. The third major element of disparity was in regard to the expenditure on and ownership of durable consumer goods which are relatively higher in urban areas. Lastly, expenditure on education by estate spending units was found to be significantly below that of urban and rural households.

In the above discussion an attempt was made to form broad impressions of the disparities in real income on the average

as between urban, rural and estate spending units. At this stage it would be of interest to inquire as to whether the pattern of income distribution in 1973 as between urban, rural and estate sectors represents an improvement on previous patterns. The table below attempts to indicate the changes that have taken place during the period 1963-73.

TABLE A III - 5

Changes in Inter Sectoral Distribution of  
Income 1963 - 73

	1963		1973	
Sector	% of Population	% of total Income	% of Population*	% of total Income
Urban	19.1	28.3	22.4	24.4
Non-Urban	80.9	71.7	77.6	75.6

Note: The above results have been derived from tables on distribution of total income in CFS 1963, and CFS 1973 and from DCS "Population of Sri Lanka" (1974) Table 4.2.

\* In the absence of a break up of the 1973 population, the 1971 break up was used.

The most significant features in the above table are the increase in the share of non-urban incomes although <sup>the</sup> proportion of non-urban population has declined, and the decline in the share of income in the case of the urban population even though the urban population has increased. These results indicate that economic changes have distinctly moved in favour of the rural community.

Since real income has increased by approximately 51 per cent over the period 1963-73 as against a population increase of 24.5 per cent (see Computation Sheet at the end of this Appendix), and since income distribution has moved in favour of the rural and estate communities, it follows that rural and estate real incomes should have shown favourable rates of growth, relative to the urban sector. The table below shows the rates at which non-urban income have increased and also the rates at which urban incomes have declined in real terms.

TABLE A III - 6  
Rates of Growth of Real Income 1963-73

Sector	Rate of growth of real income per spending Unit	Rate of growth of real income per capita
Urban	-1.5	-2.2
Rural	1.7	1.7
Estate	0.9	1.8

Note 1: See Computation Sheet for derivation of results.

Note 2: In the estate sector, the size of a spending unit has declined sharply from 5.69 in 1963 to 5.19 in 1973. This accounts for the wide disparity in growth rates as between columns 1 and 2.

The analysis carried out in the Computation Sheet also shows that urban per capita income has declined at the rate of 2.2 per cent in real terms and that non-urban per capita

income has increased at the rate of 2.7 per cent in real terms. The results are similar to those obtained by Karunatilake {1974 ; p. 98} in respect of income receivers. He estimated that average real income has declined at the rate of 1.8 per cent in the case of the urban sector, and that real income has increased at the rate of 2.0 per cent in the rural and estate sectors.

These results indicate that the urban household is worse off today than ten years ago in contrast to his rural counterpart whose conditions have improved. The worsening conditions of the urban population could be seen in terms of increasing numbers of the urban unemployed, worsening housing conditions in urban areas, demands for increased wages by urban workers, indebtedness amongst the middle income groups, the worst affected being the urban poor. In contrast, increased farmer incomes in the rural areas have enabled rural households to afford better housing facilities, to acquire consumer durables and to purchase a wider range of consumer goods. A comparison of the tables on "Amenities and Equipment" in CFS 1963 and CFS 1973 shows the manner in which the ownership of consumer durables has increased in the case of rural and estate households. However in a specific item like housing conditions, the situation in the estates may have worsened.

It would perhaps be beyond the scope of this study to go into further detail. We shall therefore shift our attention to another important source of inequality namely inequality within sectors. According to the CFS 1973 the patterns of income distribution in the urban, rural and estate sectors were found to be as follows:

TABLE A III - 7

Income Distribution - Urban, Rural and Estate Sectors  
(1973)

Income Group (Monthly Income)	Urban		Rural		Estate	
	Percent- age of Sp. Units	Perce- tage of Six Month Income	Perce- tage of Sp. Unit	Perce- tage of Six Month Income	Perce- tage of Sp Unit	Perce- tage of Six Month Income
Less than Rs.100	6.26	1.32	7.36	1.90	4.84	1.32
Rs.101-Rs.200	18.08	6.98	29.29	15.49	31.10	17.62
Rs.201 - 400	42.33	31.61	45.76	43.82	49.54	47.81
Rs.401 - 600	18.67	23.21	11.52	19.08	10.24	16.55
Rs.601 - 800	6.65	11.56	3.34	8.04	2.61	5.87
Rs.801 - 1000	3.42	7.59	1.34	4.04	0.37	1.04
Rs.1000 over	4.59	17.73	1.39	7.63	1.30	9.79
Total	100.00	100.00	100.00	100.00	100.00	100.00

Note: Results were derived from CFS 1973  
Tables S 580 - S 583.

As seen from the above, table, spending units in the income group Rs.201 - 400 constitute the largest proportion of spending units in the urban, rural and estate sectors. The other noteworthy feature in the above table is that the proportion receiving an

income of less than Rs.100 per month is less in the estate areas than in the rural and urban areas indicating the occurrence of acute poverty to lesser extent in estate areas.

The above table also shows the share of total income accruing to each income group. But in order to study the nature of income inequality in each sector, we shall rank the spending units by quintiles and consider the shares of income received and in particular the changes of these shares over time. The shares of income received by each quintile during 1963 as well as in 1973 are shown in the table below.

TABLE A III - 8

Percentage of Total Income received by each  
quintile of Ranked Spending Units in Urban,  
Rural and Estate Sectors (1963 & 1973)

Quintile of Sp. Unit	URBAN		RURAL		ESTATE		
	Share of total income received						
	1963	1973	1963	1973	1963	1973	
	Lowest	3.3	6.1	4.7	7.4	8.3	8.5
	Second	7.4	11.3	9.6	12.6	14.1	13.1
	Third	11.6	15.6	14.7	16.8	17.2	16.5
	Fourth	20.5	21.8	21.5	21.7	21.6	21.4
Fifth	57.2	45.2	49.5	41.5	38.8	40.5	
Total	100.0	100.0	100.0	100.0	100.0	100.0	

Source: Derived from Tables on Income Distribution by Spending Units in CFS 1963 and CFS 1973 i.e. Tables 501 in CFS 1963 and Tables 580-583 in CFS 1973.

The above table clearly shows the marked improvement in the shares of income accruing to the lowest quintiles both in the urban as well as rural sectors. This improvement is also accompanied by a significant decline of the share of income accruing to the top quintile both in urban and rural areas. As for the estate sector, the indications are that income distribution could have worsened. For, the shares received by the second and third quintiles show a decline and the share received by the top quintile shows a slight increase. Thus, the reduction of income inequality in the country as a whole during the period 1963-73 (as shown in Table 2.2) could be explained entirely in terms of the reduction in income inequality in the rural and urban areas. Lee {1976} expresses doubts that there has been a dramatic reduction in income inequality over the period 1963-73 on grounds that changes in relative prices, particularly that of food would have considerably distorted the measurement of income and advocate the use of consumption data instead. But he bases his arguments mainly on the decline of rice consumption per capita, and ignores the consumption of flour and rice substitutes which had increased over the same period. Moreover he does not establish alternative results based on constant price valuation of an entire basket of goods and conventional measures of inequality.



It would be relevant at this stage to examine what these changes mean in terms of a growth cum re-distribution process. If we regard the total real income in 1963 as 100 units, then real income in 1973 would be 151.1 units (see Computation Sheet). In the terms of the shares of income received in 1963 the urban sector received 28.25 units, the rural sector received 62.29 units, and the estate sector received 9.47 units in 1963. But in terms of the shares of income received in 1973, the 151.1 units of income were shared by the urban, rural and estate sectors, each receiving 36.85 units, 100.35 units and 13.90 units respectively. If each of these six estimates are now divided in terms of the shares of income received by the respective quintiles as shown in Table A III - 8 then the changes in real income received by each quintile in each sector could be seen from the tables A III - 9 and A III - 10.

TABLE A III - 9  
Distribution of Real Income in 1963

Quintile	Urban	Rural	Estate
Lowest	0.93	2.93	0.79
Second	2.09	5.98	1.34
Third	3.28	9.16	1.63
Fourth	5.79	13.39	2.05
Fifth	16.16	30.83	3.66
Total	28.25	62.29	9.47

Total = 100

TABLE A III - 10Distribution of Real Income in 1973

Quintile	Urban	Rural	Estate
Lowest	2.25	7.43	1.18
Second	4.16	12.64	1.82
Third	5.75	16.86	2.29
Fourth	8.03	21.78	2.97
Fifth	16.66	41.65	5.63
Total	36.85	100.35	13.90

Total = 151.1

The above tables indicate that the increases in real income have been distributed in favour of the poor. For, by adding up the gains by the 2 lower quintiles in each of the 3 sectors, it could be seen that a little less than one third of the total increase of 51.1 units of income have gone to these six segments of the population. The process that appears to have taken place is one of growth with a redistribution of incomes.

Computation Sheet

(Computations for Tables A III - 4 and A III - 5)

1. Population

	<u>Total</u>	<u>Urban</u>	<u>Non-Urban</u>
1963	10.582 m	2.016 m (19.1%)	8.566 m (80.9%)
1973	12.711 m	2.842 m (22.4%)	9.869 m (77.6%)
Percentage Increase	20.1%	41.0%	15.2%
Growth Rate of Population	2.3% p.a	4.4% p.a.	1.8% p.a.

Source: D.C.S. "Population of Sri Lanka" 1974 Table 4.2

2. National Income

	<u>G.D.P. at Current Factor Cost Prices</u> (Rs. m)	<u>G.D.P. at Const. 1959 Prices</u> (Rs. m)	<u>Private Dispo- sable Income at Constant prices</u> (Rs. m)
1963	6849	6796.7	6402
1973	13265	10426.4	9845
Percentage Increase		51.1%	53.7%
Growth Rate		4.4% p.a.	4.4% p.a.

Source: Central Bank of Ceylon

3. Shares of Income

1963	Total Income <sup>*</sup> of Sample Drawn : Rs.2.085 m CFS 1963 Table SUI. 1-00
	Total Income of Urban Sp.Units: Rs.20.588 m CFS 1963 Table SUI. 1-00
	Share of Urban Income : 28.3 per cent
1973	Total Income <sup>*</sup> of Sample Drawn : Rs.3,333 m (CFS 1973 Table S 580)
	Total Income of urban Sp.Units: Rs.0.813 m (CFS 1973 Table S 581)

Therefore share of Urban Income = 24.4 per cent

(\*Two month Income)

4) Rates of Growth of Income

		<u>1963</u>		
		<u>Urban</u> (Rs.)	<u>Rural</u> (Rs.)	<u>Estate</u> (Rs.)
(i)	Monthly Income per Sp. Unit at Current prices	316	166	177
	Monthly Income per Sp. Unit at (1959) Const. prices	319	168	179
	Size of Sp. Unit	4.13	5.33	5.69
	Monthly Income per capita	64.70	31.52	31.46
		<u>1973</u>		
(ii)	Monthly Income per Sp. Unit at Current prices	397	291	285
	Monthly Income per Sp. Unit at (1959) Const. prices	272	199	195
	Size of Sp. Unit	5.30	5.36	5.19
	Monthly Income per capita	51.32	37.13	37.57
		<u>Urban</u>	<u>Rural</u>	<u>Estate</u>
(iii)	Rate of Growth of Real Income per Sp. Unit	- 1.5%	1.7%	0.9%
	Rate of Growth of Real income per capita	- 2.2%	1.7%	1.8%
<u>Notes</u>				
1. The monthly income per Sp. Unit at current prices were derived from CFS 1963 Tables 501 2.00 - 2.03 and CFS 1973 Tables 580 - 583.				
2. Real incomes were derived by using the "Implicit Price Index", ie (G.D.P. at Current Prices)/G.D.P at Const. (1959) prices, to deflate incomes at current prices.				
3. The sizes of Sp. Units were derived from CFS 1963 Table 5 and CFS 1973 Table 4.				

Appendix IV

(To Chapter 4)

Solutions to Equations (4.20)

The set of linear simultaneous differential equations (4.20) are :

$$\dot{Y}_1 = \alpha_1 Y_1 + \gamma_1 Y_g \quad \dots (1)$$

$$\dot{Y}_2 = \beta_0 + \alpha_2 Y_1 + \beta_2 Y_2 + \gamma_2 Y_g \quad \dots (2)$$

$$\dot{Y}_g = \gamma_3 Y_g \quad \dots (3)$$

The solution to equation (3) is easily found as

$$Y_g = \bar{Y}_g e^{\gamma_3 t} \quad \dots (4)$$

Substituting for  $Y_g$  in equation (1), we get

$$\dot{Y}_1 - \alpha_1 Y_1 = \gamma_1 \bar{Y}_g e^{\gamma_3 t}$$

The Complementary Function =  $A_1 e^{\alpha_1 t}$ , where  $A_1$  is an arbitrary constant, and the

$$\text{Particular Integral} = \frac{\gamma_1 \bar{Y}_g e^{\gamma_3 t}}{\gamma_3 - \alpha_1}$$

Therefore, the General Solution is

$$Y_1 = A_1 e^{\alpha_1 t} + \frac{\gamma_1 \bar{Y}_g e^{\gamma_3 t}}{\gamma_3 - \alpha_1}$$

$$\text{At } t = 0, Y_1 = \bar{Y}_1$$

$$\text{Therefore } Y_1 = A_1 + \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1}$$

Eliminating  $A_1$  we have

$$Y_1 = \left[ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right] e^{\alpha_1 t} + \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} e^{\gamma_3 t} \quad \dots (5)$$

Using equations (4) and (5), we can now substitute for  $Y_g$  and  $Y_1$  in equation (2) to get

$$\dot{Y}_2 - \beta_2 Y_2 = \beta_0 + \alpha_2 \left[ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right] e^{\alpha_1 t} + \frac{\alpha_2 \gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} e^{\gamma_3 t} + \gamma_2 \bar{Y}_g e^{\gamma_3 t}$$

$$\text{Complementary Function} = A_2 e^{\beta_2 t}$$

$$\text{Particular Integral} = -\frac{\beta_0}{\beta_2} + \frac{\alpha_2}{\alpha_1 - \beta_2} \left[ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right] e^{\alpha_1 t} + \frac{\bar{Y}_g}{\gamma_3 - \beta_2} \left[ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right] e^{\gamma_3 t}$$

Therefore, the General Solution is

$$Y_2 = A_2 e^{\beta_2 t} - \frac{\beta_0}{\beta_2} + \frac{\alpha_2}{\alpha_1 - \beta_2} \left[ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right] e^{\alpha_1 t} + \frac{\bar{Y}_g}{\gamma_3 - \beta_2} \left[ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right] e^{\gamma_3 t}$$

Eliminating  $A_2$ , using the initial condition  $Y_2 = \bar{Y}_2$  at  $t = 0$

$$Y_2 = \left[ \bar{Y}_2 + \frac{\beta_0}{\beta_2} - \frac{\alpha_2}{\alpha_1 - \beta_2} \left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right\} - \frac{\bar{Y}_g}{\gamma_3 - \beta_2} \left\{ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right\} \right] e^{\beta_2 t} - \frac{\beta_0}{\beta_2} + \frac{\alpha_2}{\alpha_1 - \beta_2} \left\{ \bar{Y}_1 - \frac{\gamma_1 \bar{Y}_g}{\gamma_3 - \alpha_1} \right\} e^{\alpha_1 t} + \frac{\bar{Y}_g}{\gamma_3 - \beta_2} \left\{ \frac{\alpha_2 \gamma_1}{\gamma_3 - \alpha_1} + \gamma_2 \right\} e^{\gamma_3 t}$$

... (6)

Equations (4), (5) and (6) constitute the solution to the set of linear simultaneous differential equations (4.20)

Note: Consider the differential equation in  $Y_1$  we obtained by eliminating  $Y_g$  between equations (4) and (1)

Setting  $\alpha_1 = \gamma_3$  in this equation, it reads

$$\dot{Y}_1 - \gamma_3 Y_1 = \gamma_1 \bar{Y}_g e^{\gamma_3 t}$$

As before the complementary function =  $A_1 e^{\gamma_3 t}$

Substituting  $Y_1 = t \gamma_1 \bar{Y}_g e^{\gamma_3 t}$

$$\begin{aligned} \text{L.H.S} &= \gamma_1 \bar{Y}_g e^{\gamma_3 t} + t \gamma_3 \gamma_1 \bar{Y}_g e^{\gamma_3 t} - t \gamma_3 \gamma_1 \bar{Y}_g e^{\gamma_3 t} \\ &= \gamma_1 \bar{Y}_g e^{\gamma_3 t} \end{aligned}$$

Therefore  $t \gamma_1 \bar{Y}_g e^{\gamma_3 t}$  is a particular integral

Therefore, the general solution is

$$Y_1 = A_1 e^{\gamma_3 t} + t \gamma_1 \bar{Y}_g e^{\gamma_3 t}$$

From initial conditions,  $A_1 = \bar{Y}_1$

$$\text{Therefore } Y_1 = \bar{Y}_1 e^{\gamma_3 t} + t \gamma_1 \bar{Y}_g e^{\gamma_3 t} .$$

APPENDIX V  
(To Chapter 4)

Notes and Explanations regarding the  
Estimation of Parameters

(1) Estimation of the Ratios  $Y_1 : Y_2 : Y_g$

In the 1973 Survey of Consumer Finances,

Number of housing units sampled	= 5000
Number of households sampled	= 5088
Number of spending units per household	= 1.05
Mean Income per spending unit	= Rs. 3732 per annum

Therefore Mean Income per household = Rs. 3732 x 1.05 per annum  
= Rs. 3919 per annum.

Total number of housing units = 2,530,455

Therefore total number of households =  $\frac{5088}{5000} \times 2,530,455$

Therefore total household income = Rs. 3919 x  $\frac{5088}{5000} \times 2,530,455$

i.e.  $Y_1 + Y_2 = \text{Rs. } 10,100\text{m approximately.}$

Since the survey was conducted in January 1973, this estimate may be regarded as an estimate of household income during 1972.

From the Estimates of Government Revenue and Expenditure, it was found that

Government Revenue in 1972 = Rs. 2978 million

Personal Emoluments in 1972 = Rs. 1252 million

Therefore Government Revenue net of

Personal Emoluments = Rs. 1726 million.

Net Food Subsidy = Rs. 526m (C.B. estimate)

Therefore Government "Income" in the sense of equation  
(4.6) = Rs. 1200 million

i.e.  $Y_g = \text{Rs. } 1200$



Therefore  $(Y_1 + Y_2) : Y_g = 90 : 10$  approximately.

From Table 2.1, share of income of

Spending units receiving over Rs.400 per unit = 43 per cent.

i.e.  $Y_1 = 43$  per cent of  $(Y_1 + Y_2)$

Therefore  $Y_1 : Y_2 : Y_g = 38 : 52 : 10$  approximately.

It should also be noted that spending units receiving under Rs.400 per month constituted nearly 80 per cent of the total i.e. Population Ratio of Rich to Poor = 1 : 4.

## (2) Output - Capital Ratios

The first step in this exercise was to obtain a constant price series of GDP as well as of Gross Domestic Capital Formation (GDCF). Since the former was readily available, only the latter had to be computed. In computing the latter the first step was to split up the GDCF into its local and foreign components using estimates of the import content in GDCF provided in the Central Bank Reports of 1967 onwards under the Tables entitled "Direct Import Content of Gross Domestic Expenditure". Then GDCF at constant prices was derived as shown in the table below.

Table A - V - 1

Estimation of G.D.C.F at Constant Prices

Year	Local Component of GDCF * (Ic) (Rs.m) (1)	Ind. Taxes less Subsidies * (Current Price ) (Rs.m) (2)	G.D.E at current prices * (Rs.m) (3)	Ind. Tax ÷ G.D.E = r (4)	Ic(1-r) (Rs.m) (5)	Ic(1-r) deflated by factor cost deflator (Rs.m) (6)	Import content of GDCF * (If) (Rs.m) (7)	Import content deflated by Import Price Index * (8)	(9)=(6)+(8) G.D.C.F. at 1959 constan prices (Rs.m)
1959	749	383	6576	0.0582	705	705	364	364	1069
1960	656	408	6849	0.0596	617	617	322	295	912
1961	778	384	6876	0.0558	735	748	324	318	1066
1962	775	491	7069	0.0695	721	744	305	366	1110
1963	850	485	7510	0.0646	795	807	310	349	1156
1964	820	485	7958	0.0609	770	777	293	168	945
1965	798	529	8051	0.0657	746	753	215	126	879
1966	870	629	8627	0.0729	807	818	325	195	1013
1967	1031	754	9294	0.0811	947	940	346	187	1127
1968	1295	664	11035	0.0602	1217	1096	404	190	1286
1969	1642	900	12423	0.0724	1523	1317	611	260	1577
1970	2056	1109	13060	0.0849	1881	1570	498	182	1752
1971	1905	937	12888	0.0727	1767	1453	344	126	1597
1972	1737	981	13814	0.0710	1614	1273	381	172	1445
1973	2093	1661	16876	0.0984	1887	1289	537	207	1496
1974	2722	1624	22610	0.0718	2527	1372	418	109	1481

The estimates of GDCF at (1959) constant prices, so obtained as well as the GDP estimates (already available in C.B Reports) were then tabulated and 5 year moving averages were computed in order to find the trend in the capital output ratios. However, the capital-output ratio was estimated for the purpose of the present study in terms of the average annual increase of GDP and the average annual GDCF as shown in Table A - V - 2.

Definitions: Ic=Local Component of GDCF, If = Import Content of GDCF

\*Sources: Central Bank of Ceylon Annual Reports

Table A - V - 2

Estimation of Capital - Output Ratio

Year	G.D.C.F. at(1959) Constant Prices  (Rs. Billion)	G.D.P. at (1959) factor cost  (Rs. Billion)	G.D.C.F. 5 Year Moving Average $I_t$ (Rs. Billion)	G.D.P. 5 Year Moving Average $Y_t$ (Rs. Billion)	$\Delta Y = Y_t - Y_{t-1}$	Capital- Output Ratio  $= \frac{I_t}{Y_t - Y_{t-1}}$
1959	1.069	5.930				
1960	0.912	6.332				
1961	1.066	6.465	1.0626	6.4876		
1962	1.110	6.760	1.0378	6.7806	0.2936	3.6192
1963	1.156	6.951	1.0312	7.0276	0.2470	4.2016
1964	0.945	7.397	1.0206	7.3054	0.2778	3.7120
1965	0.879	7.565	1.0249	7.6044	0.2990	3.4134
1966	1.013	7.854	1.0500	8.0016	0.3972	2.5780
1967	1.127	8.255	1.1764	8.3960	0.3944	2.6623
1968	1.286	8.937	1.3510	8.8368	0.4408	2.6688
1969	1.577	9.369	1.4642	9.2956	0.4276	3.1595
1970	1.752	9.771	1.5278	9.5942	0.2986	4.9035
1971	1.579	9.792	1.5698	9.9634	0.3692	4.1381
1972	1.445	10.102				
1973	1.496	10.426				

Average Annual Increase in  $Y_t$  (G.D.P) during the period 1962 - 1976  

$$= \frac{\sum \Delta Y}{10} = 0.3445$$

Average Annual  $I_t$  (Investment) during the period 1961 to 1970  

$$= \frac{\sum I_t}{10} = 1.175$$

Therefore Average Incremental Capital - Output Ratio = 3.411

Therefore Average Incremental Output-Capital Ratio = 0.2932

(Note: Same results could be obtained by considering total increase in GDP and total investment)

<sup>1</sup> Source: Central Bank of Ceylon, Annual Reports

Although the aggregate capital-output ratio was thus derived, firm data was not available for the derivation of this parameter for the various sectors, i.e. the modern sector, the government sector and the traditional sector. In these circumstances, the use of 'judgement' was inevitable. Accordingly the output-capital ratio in government was assumed to be 0.25 considering the heavy infra-structural components in government enterprises.<sup>1</sup> The parameters  $a_1$  and  $b_2$  were both set at 0.30 (in Set 2). But in view of higher efficiencies that could prevail in investments which employ only highly skilled labour or investment abroad  $b_1$  was set at 0.32.

### (3). Wage and Profitability Parameters

The first source of information for this computation was "A Framework of Economic Statistics in Sri Lanka with special reference to Employment and Income Distribution" by Pyatt et al. According to the Table on summarised social accounts, the total of wages in the Private Sector was Rs.4114m and the total value added was Rs.9911m. Thus the share of wages, (assumed as wage income of poor) was taken as 0.4.

The balance proportion of 0.6 was taken to consist of salaries and management fees amounting to 0.1 and pure profits amounting to 0.5. Thus

$$w_{11} = 0.1$$

$$w_{21} = 0.4$$

$$p_1 = 0.5$$

In the case of the government sector, the total of personal emolu-

<sup>1</sup> This would have the effect of lowering the productivity of capital

ments was earlier noted to be Rs.1250m approximately in 1972. Together with the wage bills in Corporations and Co-operative enterprises, the total wage bill was estimated to be in the region of Rs.4500m.

Therefore  $(w_{1g} + w_{2g}) : p_g = 4500 : 1200 = 4 : 1$  approximately. Tables II J 2 and II J 3 of the C.B. report 1972 gives information on Employment in Government institutions and in Corporations. By considering the "Administrative Technical and Professional Offices of staff rank" as well as certain categories of "Subordinate employees" to constitute the "Rich" in Government Services and by assuming plausible monthly incomes it was estimated that  $w_{1g} : w_{2g} = 1 : 3$

$$\text{Therefore } w_{1g} : w_{2g} : p_g = 2 : 6 : 2$$

#### (4) Savings Ratios

The basic source of information for the estimate of the marginal savings rate was a mimeographed paper by P.N. Radhakrishnan entitled "Aggregate Savings and its components - Trends, Prospects and Implications - Sri Lanka" on the basis of time series data covering the period 1959-1974, he estimated the aggregate savings function to be

$$S_t = 0.17 Y_t - 288.12$$

(0.014)

In the absence of cross-section estimates, the time series estimate of 0.17 was assumed to be the marginal savings rate of the poor. The average savings rate of rich was assumed to be fixed at 20 per cent. Thereafter, the constant term  $s_0$  in the savings function of the poor was calculated as follows:

Savings function of the Poor:

$$S_2 = 0.17 Y_2 + s_0$$

$$\text{Therefore average savings rate} = \frac{s_2}{Y_2} = 0.17 + \frac{s_0}{Y_2}$$

$$\text{Since } Y_2 = 38$$

$$\text{Average savings rate of poor} = 0.17 + \frac{s_0}{38}$$

$$\text{Population ratio of Rich to Poor} = 1 : 4$$

$$\text{Therefore weighted average savings rate} = 1/5 (0.2 + 4(0.17 + \frac{s_0}{38}))$$

The weighted average was assumed to be 10 per cent since the saving rate during the period 1972/73 was estimated to be around this figure.

$$\text{i.e. } 1/5 (0.2 + 0.4 (0.17 + \frac{s_0}{38})) = 0.1$$

$$\text{Therefore } s_0 = 3.61$$

figure of

A higher  $\frac{1}{4}$  was taken to be the estimate of  $s_0$ .

Appendix VI

(To Chapter 4)

Solutions to Equations (4.52)

As in the case of equations (4.43) in Section 4.8, the values of the basic parameters used are the same as those of Set 1, except for  $b_2 = 0.28$  and  $s_2 = 0.1$ . In terms of these values,

$$\alpha_1 = 0.0472, \quad \gamma_1 = 0.005$$

$$\alpha_2 = 0.0144, \quad \beta_2 = 0.028, \quad \beta_0 = -1.12,$$

$$\gamma_2 = 0.005, \quad \gamma_3 = 0.005, \quad \eta_1 = 1$$

The transformation (4.51) then yields values for  $\alpha'_1$ ,  $\gamma'_1$ ,  $\alpha'_2$  etc. for each value of  $\lambda$ , and the coefficients of equation (4.52) could thus be calculated.

Case I       $\lambda = 0.05$

Equations (4.52) read:

$$\dot{Y}_1 = 0.04509 Y_1 + 0.00475 Y_g$$

$$\dot{Y}_2 = 0.01729 Y_1 - 1.12 + 0.028 Y_2 + 0.01522 Y_g$$

$$\dot{Y}_g = 0.00503 Y_g + 0.000526 Y_1 \quad \text{--- (1)}$$

Let  $\mathcal{L}(Y_1) = y_1$  and  $\mathcal{L}(Y_2) = y_2$

Applying the Laplace Transform to the first and third of equations (1)<sup>1</sup> and solving for  $y_1$  and  $y_2$  we get:

$$\begin{aligned} y_1 &= \frac{38(s - 0.00503) + 0.0475}{s^2 - 0.0501s + 0.0002243} \\ y_g &= \frac{0.0200 + 10(s - 0.04509)}{s^2 - 0.0501s + 0.0002243} \end{aligned} \quad \text{--- (2)}$$

These equations can be put into the partial fraction form:

$$\begin{aligned} y_1 &= \frac{39.125}{(s - 0.04516)} - \frac{1.125}{(s - 0.00497)} \\ y_g &= \frac{0.515}{(s - 0.0415)} + \frac{9.485}{(s - 0.005)} \end{aligned} \quad \text{--- (3)}$$

Using the second property defined by equations (4.54), i.e. that  $e^{at} = \frac{1}{s-a}$  or conversely that  $e^{at} = -1 \frac{1}{s-a}$ , the solutions to  $Y_1$  and  $Y_g$  are:

$$\begin{aligned} Y_1 &= 39.125 e^{0.04516t} - 1.125 e^{0.00497t} \\ Y_g &= 0.515 e^{0.04516t} + 9.485 e^{0.00497t} \end{aligned} \quad \text{--- (4)}$$

<sup>1</sup> As defined by equations (4.54),

$$\mathcal{L}\{\dot{F}\} = s f(s) - F(0)$$

Therefore, in place of equations (4.55) we get

$$\begin{aligned} sy_1 - \bar{Y}_1 &= 0.04509 y_1 + 0.00475 y_g \\ sy_g - \bar{Y}_g &= 0.00503 y_g + 0.000526 y_1, \\ \text{where } \bar{Y}_1 &= 38 \text{ and } \bar{Y}_g = 10 \end{aligned}$$

These equations could be easily solved for  $y_1$  and  $y_g$ .

Alternatively, the values of  $\alpha'_1$ ,  $\gamma'_1$ , etc. and of  $\bar{Y}_1$  and  $\bar{Y}_g$  could be directly substituted in equations (4.56)



Substituting for  $Y_1$  and  $Y_g$  in the second of equations (1),

$$\dot{Y}_2 - 0.028 Y_2 = -1.12 + 0.6841 e^{0.04516t} + 0.1252 e^{0.00497t}$$

$$\text{Therefore, } Y_2 = Ae^{0.028t} + 40 + 39.866 e^{0.0451t} - 5.436 e^{0.00497t}$$

$$\text{At } t = 0, \quad 52 = A + 74.430$$

Therefore,  $A = 22.430$  and

$$Y_2 = 40 + 39.866 e^{0.0451t} - 22.430 e^{0.028t} - 5.436 e^{0.00497t}$$

--- (5)

$$\text{At } t = 10, \quad Y_1 = 60.276$$

$$Y_2 = 67.194$$

$$Y_g = 10.771$$

$$\text{Therefore, } Y_1 + Y_2 + Y_g = 138.247,$$

$$Y_1 + Y_2 = 127.47 \quad \text{and}$$

$$Y_1/Y_2 = 0.897$$

Case II       $\lambda = 0.10$

Equations (4.52) read:

$$\dot{Y}_1 = 0.04298 Y_1 + 0.0045 Y_g$$

$$\dot{Y}_2 = 0.0203 Y_1 - 1.12 + 0.028 Y_2 + 0.01544 Y_g$$

$$\dot{Y}_g = 0.00505 Y_g + 0.001086 Y_1 \quad \text{--- (1)}$$

Applying the Laplace Transform to the first and the third of equations (1) and solving for  $y_1$  and  $y_2$  we get:

$$y_1 = \frac{38(s - 0.00505) + 0.045}{s^2 - 0.04803s + 0.0002122}$$

$$y_g = \frac{0.04085 + 10(s - 0.04298)}{s^2 - 0.04803s + 0.0002122} \quad \text{--- (2)}$$

These equations can be put into the partial fraction form:

$$y_1 = \frac{39.059}{(s - 0.04311)} - \frac{1.059}{(s - 0.00493)}$$

$$y_g = \frac{1.104}{(s - 0.0411)} + \frac{8.896}{(s - 0.00493)} \quad \text{--- (3)}$$

As before, the inverse Laplace Transform yields

$$Y_1 = 39.059 e^{0.04311t} - 1.059 e^{0.00493t}$$

$$Y_g = 8.896 e^{0.00493t} + 1.104 e^{0.04311t} \quad \text{--- (4)}$$

Substituting for  $Y_1$  and  $Y_g$  in the second of equations (1),

$$\dot{Y}_2 - 0.028 Y_2 = -1.12 + 0.8099 e^{0.04311t} + 0.1159 e^{0.00493t}$$

$$\text{Therefore, } Y_2 = A e^{0.028t} + 40 + 53.600 e^{0.04311t} - 5.024 e^{0.00493t}$$

--- (5)

$$\text{At } t = 0, \quad 52 = A + 88.876$$

Therefore,  $A = -36.576$  and

$$Y_2 = 40 - 36.576 e^{0.028t} + 53.600 e^{0.04311t} - 5.024 e^{0.00493t}$$

$$\text{At } t = 10, \quad Y_1 = 58.997$$

$$Y_2 = 68.815$$

$$Y_g = 11.045$$

$$\text{Therefore, } Y_1 + Y_2 + Y_g = 138.857$$

$$Y_1 + Y_2 = 127.812 \text{ and}$$

$$Y_1/Y_2 = 0.857$$

Case III       $\lambda = 0.15$

Equations (4.52) read:

$$\dot{Y}_1 = 0.04087 Y_1 + 0.00425 Y_g$$

$$\dot{Y}_2 = 0.0235 Y_1 - 1.12 + 0.028 Y_2 + 0.01567 Y_g$$

$$\dot{Y}_g = 0.00508 Y_g + 0.001683 Y_1 \quad \text{--- (1)}$$

Applying the Laplace Transform to the first and third of equations (1) and solving for  $y_1$  and  $y_2$  we get:

$$y_1 = \frac{38(s - 0.00508) + 0.0425}{s^2 - 0.04595s + 0.0002004}$$

$$y_g = \frac{0.06395 + 10(s - 0.04195)}{s^2 - 0.04595s + 0.0002004} \quad \text{--- (2)}$$

These equations can be put into the partial fraction form:

$$y_1 = \frac{38.964}{(s - 0.04107)} - \frac{0.964}{(s - 0.00488)}$$

$$y_g = \frac{1.822}{(s - 0.04107)} + \frac{8.178}{(s - 0.00488)} \quad \text{--- (3)}$$

Therefore, the inverse Laplace Transform yields

$$Y_1 = 38.964 e^{0.04107t} - 0.964 e^{0.00488t}$$

$$Y_g = 1.822 e^{0.04107t} + 8.178 e^{0.00488t} \quad \text{--- (4)}$$

Substituting for  $Y_1$  and  $Y_g$  in the second of equations (1)

$$\dot{Y}_2 - 0.028 Y_2 = -1.12 + 0.9442 e^{0.04107t} + 0.1055 e^{0.00488t}$$

$$\text{Therefore, } Y_2 = A e^{0.028t} + 40 + 72.242 e^{0.04107t} - 4.563 e^{0.00488t}$$

$$\text{--- (5)}$$

$$\text{At } t = 0, \quad 52 = A + 107.679$$

Therefore,  $A = -55.679$  and

$$Y_2 = 40 + 72.242 e^{0.04107t} - 55.679 e^{0.028t} - 4.563 e^{0.00488t}$$

$$\text{At } t = 10, \quad Y_1 = 57.741$$

$$Y_2 = 70.470$$

$$Y_g = 11.365$$

$$\text{Therefore, } Y_1 + Y_2 + Y_g = 139.576$$

$$Y_1 + Y_2 = 128.211, \text{ and}$$

$$Y_1/Y_2 = 0.819$$

Case IV       $\lambda = 0.20$

Equations (4.52) read:

$$\dot{Y}_1 = 0.03876 Y_1 + 0.004 Y_g$$

$$\dot{Y}_2 = 0.0268 Y_1 - 1.12 + 0.028 Y_2 + 0.01589 Y_g$$

$$\dot{Y}_g = 0.00511 Y_g + 0.002326 Y_1 \quad \text{--- (1)}$$

Applying the Laplace Transform to the first and third of equations (1) and solving for  $y_1$  and  $y_g$  we get:

$$y_1 = \frac{38 (s - 0.00511) + 0.04}{s^2 - 0.04387s + 0.000189}$$

$$y_g = \frac{0.0884 + 10 (s - 0.03876)}{s^2 - 0.04387s + 0.000189} \quad \text{--- (2)}$$

These equations can be put into the partial fraction form:

$$y_1 = \frac{38.881}{(s - 0.03903)} - \frac{0.881}{(s - 0.00485)}$$

$$y_g = \frac{2.665}{(s - 0.03903)} + \frac{7.335}{(s - 0.00485)} \quad \text{--- (3)}$$

Therefore, the inverse Laplace Transform yields

$$Y_1 = 38.881 e^{0.03903t} - 0.881 e^{0.00485t}$$

$$Y_g = 2.665 e^{0.03903t} + 7.335 e^{0.00485t} \quad \text{--- (4)}$$

Substituting for  $Y_1$  and  $Y_g$  in the second of equations (1)

$$\dot{Y}_2 - 0.028 Y_2 = -1.12 + 1.084 e^{0.03903t} + 0.09294 e^{0.00485t}$$

$$\text{Therefore, } Y_2 = A e^{0.028t} + 40 + 98.277 e^{0.03903t} - 4.015 e^{0.00485t}$$

--- (5)

$$\text{At } t = 0, \quad 52 = 134.262 + A$$

Therefore,  $A = -82.262$  and

$$Y_2 = 40 + 98.277 e^{0.03903t} - 82.262 e^{0.028t} - 4.015 e^{0.00485t}$$

$$\text{At } t = 10, \quad Y_1 = 56.519$$

$$Y_2 = 72.138$$

$$Y_g = 11.637$$

$$\text{Therefore, } Y_1 + Y_2 + Y_g = 140.294$$

$$Y_1 + Y_2 = 128.657, \text{ and}$$

$$Y_1/Y_2 = 0.783$$

Case V       $\lambda = 0.25$

Equations (4.52) read:

$$\dot{Y}_1 = 0.03665 Y_1 + 0.00375 Y_g$$

$$\dot{Y}_2 = 0.0303 Y_1 - 1.12 + 0.028 Y_2 + 0.01611 Y_g$$

$$\dot{Y}_g = 0.00514 Y_g + 0.003026 Y_1$$

--- (1)

Applying the Laplace Transform to the first and third of equations (1) and solving for  $y_1$  and  $y_2$  we get:

$$y_1 = \frac{38 (s - 0.00514) + 0.0375}{s^2 - 0.04179 s + 0.000177}$$

$$y_g = \frac{0.1150 + 10 (s - 0.03665)}{s^2 - 0.04179s + 0.000177} \quad \text{--- (2)}$$

These equations can be put into the partial fraction form:

$$y_1 = \frac{38.751}{(s - 0.03701)} - \frac{0.751}{(s - 0.00479)}$$

$$y_g = \frac{3.681}{(s - 0.03701)} + \frac{6.319}{(s - 0.00479)} \quad \text{--- (3)}$$

Therefore, the inverse Laplace Transform yields

$$Y_1 = 38.751 e^{0.03701t} - 0.751 e^{0.00479t}$$

$$Y_g = 3.681 e^{0.03701t} + 6.319 e^{0.00479t} \quad \text{--- (4)}$$

Substituting for  $Y_1$  and  $Y_g$  in the second of equations (1),

$$\dot{Y}_2 - 0.028 Y_2 = -1.12 + 1.233 e^{0.03701t} + 0.07905 e^{0.00479t}$$

$$\text{Therefore, } Y_2 = A e^{0.028t} + 40 + 136.848 e^{0.03701t} - 3.406 e^{0.00479t}$$

--- (5)

$$\text{At } t = 0, \quad 52 = 173.442 + A$$

Therefore,  $A = -121.442$ , and

$$Y_2 = 40 + 136.848 e^{0.03701t} - 121.442 e^{0.028t} - 3.406 e^{0.00479t}$$

$$\text{At } t = 10, \quad Y_1 = 55.319$$

$$Y_2 = 73.883$$

$$Y_g = 11.959$$

$$\text{Therefore, } Y_1 + Y_2 + Y_g = 141.161$$

$$Y_1 + Y_2 = 129.202, \text{ and}$$

$$Y_1/Y_2 = 0.749$$



APPENDIX VII

(To Chapter 5)

Solutions to Equations (5.21)

The set of linear simultaneous differential equations (5.21) are:

$$\dot{Y}_1 = \alpha_1 Y_1 + \gamma_1 Y_g + \eta_1 F_t$$

$$\dot{Y}_2 = \beta_0 + \alpha_2 Y_1 + \beta_2 Y_2 + \gamma_2 Y_g + \eta_2 F_t + \nu_2 \dot{Y}_1 + \epsilon S_t$$

$$\dot{Y}_g = \alpha_3 Y_1 + \gamma_3 Y_g + \eta_3 F_t + \gamma_3 \dot{Y}_1 - \epsilon S_t$$

In order to solve the above equations we shall first select the first and the third equations and write them in the form

$$(D - \alpha_1)Y_1 - \gamma_1 Y_g = \eta_1 F_t$$

$$(\nu_3 D + \alpha_3)Y_1 - (D - \gamma_3)Y_g = \epsilon S_t - \eta_3 F_t \quad \dots \quad (1)$$

where D represents the operator  $\frac{d}{dt}$

Using the operator  $(D - \gamma_3)$  on the first equation and multiplying the second equation by  $\gamma_1$ , we get

$$(D - \alpha_1)(D - \gamma_3)Y_1 - \gamma_1 (D - \gamma_3)Y_g = \eta_1 (D - \gamma_3)F_t$$

$$\gamma_1 (\nu_3 D + \alpha_3)Y_1 - \gamma_1 (D - \gamma_3)Y_g = \gamma_1 \epsilon S_t - \eta_3 \gamma_1 F_t \quad \dots \quad (2)$$

Eliminating  $Y_g$  we get

$$\left[ (D - \alpha_1)(D - \gamma_3) - \gamma_1 \nu_3 D - \gamma_1 \alpha_3 \right] Y_1 = \eta_1 (D - \gamma_3)F_t - \gamma_1 \epsilon S_t + \eta_3 \gamma_1 F_t$$

$$\text{i.e. } \left[ D^2 - (\alpha_1 + \gamma_3 + \gamma_1 \nu_3)D + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3) \right] Y_1 = \eta_1 (D - \gamma_3)F_t - \epsilon \gamma_1 S_t + \eta_3 \gamma_1 F_t \quad \dots \quad (3)$$

The auxiliary equation is

$$\lambda^2 - (\alpha_1 + \gamma_3 + \gamma_1 \nu_3)\lambda + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3) = 0 \quad \dots \quad (4)$$

Should this equation have complex roots, the solution obtained in respect of  $Y_1$  will be oscillatory. However, as shown below, the roots of equation (4) are real and distinct.

The discriminant of this equation is

$$\begin{aligned}\Delta &= (\alpha_1 + \gamma_3 + \gamma_1 \nu_3)^2 - 4(\alpha_1 \gamma_3 - \gamma_1 \alpha_3) \\ &= \alpha_1^2 + 2\alpha_1 \gamma_3 + \gamma_3^2 + 2\gamma_1 \nu_3(\alpha_1 + \gamma_3) + \gamma_1^2 \nu_3^2 - 4\alpha_1 \gamma_3 + 4\gamma_1 \alpha_3 \\ &= (\alpha_1 - \gamma_3)^2 + \gamma_1^2 \nu_3^2 + 2\gamma_1 [\nu_3(\alpha_1 + \gamma_3) + 2\alpha_3] \quad \dots (5)\end{aligned}$$

Clearly,  $(\alpha_1 - \gamma_3)^2 \geq 0$ ,  $\gamma_1^2 \nu_3^2 > 0$ . Further, from the definition of the coefficients in equation (1),  $\gamma_1$ ,  $\nu_3$ ,  $\alpha_1$ ,  $\gamma_3$  and  $\alpha_3$  are positive

$$\text{Therefore } \Delta > 0$$

The roots of equation (4) are therefore real and distinct.

Suppose we denote the two roots by  $\lambda_1$  and  $\lambda_2$  defined by:

$$\begin{aligned}\lambda_1 &= \frac{(\alpha_1 + \gamma_3 + \gamma_1 \nu_3) + \sqrt{(\alpha_1 + \gamma_3 + \gamma_1 \nu_3)^2 - 4(\alpha_1 \gamma_3 - \alpha_3 \gamma_1)}}{2} \\ \text{and } \lambda_2 &= \frac{(\alpha_1 + \gamma_3 + \gamma_1 \nu_3) - \sqrt{(\alpha_1 + \gamma_3 + \gamma_1 \nu_3)^2 - 4(\alpha_1 \gamma_3 - \alpha_3 \gamma_1)}}{2} \quad \dots (6)\end{aligned}$$

then the complementary function is given by

$$Y_1 = A e^{\lambda_1 t} + B e^{\lambda_2 t} \quad \dots (7)$$

where A and B are constants to be determined from initial conditions.

The particular integral of equation (3) is given by

$$P.I = \frac{1}{D^2 - (\alpha_1 + \gamma_3 + \gamma_1 \gamma_3)D + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)} \left\{ \eta_1 (D - \gamma_3) F_t - \epsilon \gamma_1 S_t + \eta_3 \gamma_1 F_t \right\} \dots (8)$$

$$\text{Since } F_t = F_0 e^{\delta t}, \quad S_t = S_0 e^{\epsilon t},$$

$$\begin{aligned} P.I &= \frac{1}{D^2 - (\alpha_1 + \gamma_3 + \gamma_1 \gamma_3)D + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)} \left[ \{ \eta_1 (\delta - \gamma_3) + \eta_3 \gamma_1 \} F_0 e^{\delta t} - \epsilon \gamma_1 S_0 e^{\epsilon t} \right] \\ &= \frac{\{ \eta_1 (\delta - \gamma_3) + \eta_3 \gamma_1 \} F_0 e^{\delta t}}{\delta^2 - (\alpha_1 + \gamma_3 + \gamma_1 \gamma_3)\delta + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)} - \frac{\epsilon \gamma_1 S_0 e^{\epsilon t}}{\epsilon^2 - (\alpha_1 + \gamma_3 + \gamma_1 \gamma_3)\epsilon + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)} \\ &= \mu_1 F_0 e^{\delta t} - \mu_2 S_0 e^{\epsilon t} \dots (9) \end{aligned}$$

$$\text{where } \mu_1 = \frac{\{ \eta_1 (\delta - \gamma_3) + \eta_3 \gamma_1 \}}{\delta^2 - (\alpha_1 + \gamma_3 + \gamma_1 \gamma_3)\delta + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)}$$

$$\text{and } \mu_2 = \frac{\epsilon \gamma_1}{\epsilon^2 - (\alpha_1 + \gamma_3 + \gamma_1 \gamma_3)\epsilon + (\alpha_1 \gamma_3 - \gamma_1 \alpha_3)}$$

Therefore the complete solution of equation (3) is

$$Y_1 = A e^{\lambda_1 t} + B e^{\lambda_2 t} + \mu_1 F_0 e^{\delta t} - \mu_2 S_0 e^{\epsilon t} \dots (10)$$

When  $t = 0$   $Y_1 = \bar{Y}_1$  (initial income of the rich)

$$\text{Therefore } \bar{Y}_1 = A + B + \mu_1 F_0 - \mu_2 S_0 \dots (11)$$

Substituting for  $Y_1$  in the first of equations (1),

$$\begin{aligned} \gamma_1 Y_g &= (\lambda_1 - \alpha_1) A e^{\lambda_1 t} + (\lambda_2 - \alpha_1) B e^{\lambda_2 t} + (\delta - \alpha_1) \mu_1 F_0 e^{\delta t} \\ &\quad - (\epsilon - \alpha_1) \mu_2 S_0 e^{\epsilon t} - \eta_1 F_0 e^{\delta t} \dots (12) \end{aligned}$$

When  $t = 0$   $Y_g = \bar{Y}_g$  (initial government income)

$$\text{Therefore } \gamma_1 \bar{Y}_g = (\lambda_1 - \alpha_1) A + (\lambda_2 - \alpha_1) B + \{ (\delta - \alpha_1) \mu_1 - \eta_1 F_0 - (\epsilon - \alpha_1) \mu_2 S_0 \} \dots (13)$$

Solving equations (11) and (13) we get:

$$A = \frac{\gamma_1 \bar{Y}_g - (\lambda_2 - \alpha_1) \bar{Y}_1 + [\eta_1 - (\delta - \lambda_2) \mu_1] F_0 - (\epsilon - \lambda_2) \mu_2 S_0}{(\lambda_1 - \lambda_2)}$$

$$\text{and } B = \frac{(\lambda_1 - \alpha_1) \bar{Y}_1 - \gamma_1 \bar{Y}_g + \mu_2 S_0 (\lambda_1 - \epsilon) + F_0 [\mu_1 (\delta - \lambda_1) - \eta_1]}{(\lambda_1 - \lambda_2)} \dots (14)$$

In order to solve for  $Y_2$ , we shall in the first instance take the second of equations (5.22), substitute for  $Y_1$  from the first of equations of (5.22) and re-arrange terms to get :

$$\dot{Y}_2 - \beta_2 Y_2 = \beta_0 + (\alpha_2 + \gamma_2 \alpha_1) Y_1 + (\gamma_2 + \gamma_2 \gamma_1) Y_g + (\eta_2 + \gamma_2 \eta_1) F_t + S_t \dots (15)$$

The complementary function is  $Y_2 = C e^{\beta_2 t}$  where  $C$  is a constant to be determined from the initial conditions.

The Particular Integral is given by:

$$\begin{aligned} \text{P.I} = & -\frac{\beta_0}{\beta_2} + (\alpha_2 + \gamma_2 \alpha_1) \frac{Y_1}{(D - \beta_2)} + (\gamma_2 + \gamma_2 \gamma_1) \frac{Y_g}{(D - \beta_2)} + (\eta_2 + \gamma_2 \eta_1) \frac{F_0 e^{\delta t}}{(D - \beta_2)} \\ & + \frac{\epsilon S_0 e^{\epsilon t}}{(D - \beta_2)} \dots (16) \end{aligned}$$

Substituting for  $Y_1$  and  $Y_g$  from equations (10) and (12) we get:

$$\begin{aligned} \text{P.I} = & -\frac{\beta_0}{\beta_2} + (\alpha_2 + \gamma_2 \alpha_1) \left[ \frac{A e^{\lambda_1 t}}{\lambda_1 - \beta_2} + \frac{B e^{\lambda_2 t}}{\lambda_2 - \beta_2} + \frac{\mu_1 F_0 e^{\delta t}}{\delta - \beta_2} - \frac{\mu_2 S_0 e^{\epsilon t}}{\epsilon - \beta_2} \right] \\ & + \frac{(\gamma_2 + \gamma_2 \gamma_1)}{\gamma_1} \left[ \frac{(\lambda_1 - \alpha_1) A e^{\lambda_1 t}}{(\lambda_1 - \beta_2)} + \frac{(\lambda_2 - \alpha_1) B e^{\lambda_2 t}}{(\lambda_2 - \beta_2)} \right] \end{aligned}$$

$$\begin{aligned}
& + \left[ \frac{\{(\delta - \alpha_1)\mu_1 - \eta_1\} F_0 e^{\delta t}}{(\delta - \beta_2)} - \frac{(\epsilon - \alpha_1)\mu_2 S_0 e^{\epsilon t}}{\epsilon - \beta_2} \right] \\
& + (\eta_2 + \gamma_2 \eta_1) \frac{F_0 e^{\delta t}}{(\delta - \beta_2)} + \frac{S_0 e^{\epsilon t}}{(\epsilon - \beta_2)} \quad \dots \quad (17)
\end{aligned}$$

Therefore the complete solution of  $Y_2$  is:

$$\begin{aligned}
Y_2 = & C e^{\beta_2 t} \frac{\beta_0}{\beta_2} + (\alpha_2 + \gamma_2 \alpha_1) \left[ \frac{A e^{\lambda_1 t}}{\lambda_1 - \beta_2} + \frac{B e^{\lambda_2 t}}{\lambda_2 - \beta_2} + \frac{\mu_1 F_0 e^{\delta t}}{\delta - \beta_2} - \frac{\mu_2 S_0 e^{\epsilon t}}{\epsilon - \beta_2} \right] \\
& + \frac{(\gamma_2 + \gamma_2 \gamma_1)}{\gamma_1} \left[ \frac{(\lambda_1 - \alpha_1) A e^{\lambda_1 t}}{(\lambda_1 - \beta_2)} + \frac{(\lambda_2 - \alpha_1) B e^{\lambda_2 t}}{(\lambda_2 - \beta_2)} + \frac{\{(\delta - \alpha_1)\mu_1 - \eta_1\} F_0 e^{\delta t}}{(\delta - \beta_2)} \right. \\
& \left. - \frac{(\epsilon - \alpha_1)\mu_2 S_0 e^{\epsilon t}}{(\epsilon - \beta_2)} \right] + (\eta_2 + \gamma_2 \eta_1) \frac{F_0 e^{\delta t}}{(\delta - \beta_2)} + \frac{S_0 e^{\epsilon t}}{(\epsilon - \beta_2)} \quad \dots \quad (18)
\end{aligned}$$

where from initial conditions,  $C$  is given by

$$\begin{aligned}
C = & \bar{Y}_2 + \frac{\beta_0}{\beta_2} - (\alpha_2 + \gamma_2 \alpha_1) \left[ \frac{A}{\lambda_1 - \beta_2} + \frac{B}{\lambda_2 - \beta_2} + \frac{\mu_1 F_0}{\delta - \beta_2} - \frac{\mu_2 S_0}{\epsilon - \beta_2} \right] \\
& - \frac{(\gamma_2 + \gamma_2 \gamma_1)}{\gamma_1} \left[ \frac{(\lambda_1 - \alpha_1) A}{(\lambda_1 - \beta_2)} + \frac{(\lambda_2 - \alpha_1) B}{(\lambda_2 - \beta_2)} + \frac{\{(\delta - \alpha_1)\mu_1 - \eta_1\} F_0}{(\delta - \beta_2)} - \frac{(\epsilon - \alpha_1)\mu_2 S_0}{(\epsilon - \beta_2)} \right] \\
& - (\eta_2 + \gamma_2 \eta_1) \frac{F_0}{(\delta - \beta_2)} - \frac{\epsilon S_0}{(\epsilon - \beta_2)} \quad \dots \quad (19)
\end{aligned}$$

Thus the complete solution to the system of equations (5.22) is given by equations (10), (12) and (18), where the constants A, B and C are determined by equations (14) and (19).

APPENDIX VIII

(To Chapter 7)

Explanatory Note No.1 to Chapter 7

The values of parameters used are essentially the same as those estimated for Chapter 4, except for the parameters  $s_2$  which is taken to be 0.1

That is

$$a_1 = 0.3, \quad \beta_2 = 0.28 \text{ (same as } b_2 \text{ in Ch.4)} \quad a_g = 0.25$$

$$w_{11} = 0.1 \quad w_{21} = 0.4 \quad p_1 = 0.5$$

$$w_{2g} = 0.8 \quad p_g = 0.2$$

$$\text{Now } \alpha_1 = a_1(w_{11} + p_1), \quad \alpha_2 = w_{21} a_1 \quad \beta_2 = b_2, \quad \gamma_2 = w_{2g} a_g$$

$$\text{and } \gamma_3 = p_g a_g$$

$$\text{Therefore } \alpha_1 = 0.18, \quad \alpha_2 = 0.12, \quad \beta_2 = 0.28 \quad \gamma_2 = 0.2$$

$$\text{and } \gamma_3 = 0.05$$

Since the rate of population increase has varied between 1.7 and 1.6 per cent during the past 4 or 5 years, we shall take  $r_1 = 0.015$  and  $r_2 = 0.018$ , as plausible values of these parameters. The implicit assumption here is that fertility at higher income levels is considerably low.

$$\text{Income earned by the rich} = \alpha_1 K_1$$

$$\text{But lowest income received by the rich} = Z_1$$

$$\text{Let } Z_1 \text{ be set so that } Z_1 = 0.8 \alpha_1 K_1$$

That is  $Z_1$  is so defined that at least 80 per cent of their income initially earned, is assured to them.

$$\text{Therefore savings} = s_1 Z_1 = s_1 0.8 \alpha_1 \bar{K}_1 = 0.0288 \bar{K}_1$$

$$\text{since } r_1 = 0.015, K_1 (e^{r_1 T} - 1) = 0.455 \bar{K}_1 \text{ when } T = 25$$

$$\text{Also } s_1 Z_1 T = 0.72 \bar{K}_1, \text{ when } T = 25$$

$$\text{Therefore } s_1 Z_1 T > K_1 (e^{r_1 T} - 1) \quad \dots\dots (1)$$

Clearly, this inequality will hold for values of T less than 25 years.

$$\text{Now } s_2 \beta_2 = 0.028$$

$$\text{But } r_2 = 0.018$$

$$\text{Therefore } s_2 \beta_2 > r_2 \quad \dots\dots (2)$$

$$\text{Again } \gamma_2 = 0.2$$

$$\text{But } \beta_2 (1 - s_2) = 0.28 \times 0.9 = 0.252$$

$$\text{Therefore } \gamma_2 < \beta_2 (1 - s_2) \quad \dots\dots (3)$$



Appendix IX  
(To Chapter 7)

Explanatory Note No.2 to Chapter 7

The object of this note is to examine whether the rich and the poor could reverse roles during a finite time horizon of 25 years. The best of possible circumstances which could bring about a reversal is Regime II with  $\lambda = 1$ ,  $\mu = 1$ , indicative of strong redistribution measures. In this case the relevant equations are.

$$Y_1 = Z_1$$

$$Y_2 = a_1 K_1 - Z_1 + \beta_2 K_2 + \gamma_2 K_g$$

$$Y_g = \gamma_3 K_g \quad \dots (1)$$

$$\dot{K}_1 = s_1 Z_1$$

$$\dot{K}_2 = s_2 \beta_2 K_2 + (s_g \gamma_3 K_g - I_g)$$

$$\dot{K}_g = I_g \quad \dots (2)$$

Since  $\beta_2 > \gamma_2$  (see Appendix VIII), it follows that an increment in  $K_2$  will favour a higher  $Y_2$  than an equivalent increment in  $K_g$ . Moreover, equations (2) indicate that  $K_2$  has an exponential growth in contrast to the purely linear growth of  $K_g$  (since  $I_g = \text{constant}$ ). Therefore, the accumulation of  $K_2$  ought to be faster than that of  $K_g$ .

It is clear from the second equation of (2) that  $K_2$  will be higher with lower values of  $I_g$ ,

Setting  $I_g = 0$ , equations (2) read

$$\dot{K}_1 = s_1 z_1$$

$$\dot{K}_2 = s_2 \beta_2 K_2 + s_g \gamma_3 K_g$$

$$\dot{K}_g = 0 \quad \dots (3)$$

$$\text{Therefore } K_1 = s_1 z_1 t + \bar{K}_1$$

$$K_g = \bar{K}_g \quad \dots (4)$$

Substituting for  $K_g$  in the second equation of (3)

$$\dot{K}_2 - s_2 \beta_2 K_2 = s_g \gamma_3 \bar{K}_g$$

$$\text{The solution is: } K_2 = A e^{s_2 \beta_2 t} - \frac{s_g \gamma_3 \bar{K}_g}{s_2 \beta_2}$$

where A is an arbitrary constant.

$$\text{Initially, when } t = 0, \bar{K}_2 = A - \frac{s_g \gamma_3 \bar{K}_g}{s_2 \beta_2}$$

$$\text{Therefore } K_2 = (\bar{K}_2 + \frac{s_g \gamma_3 \bar{K}_g}{s_2 \beta_2}) e^{s_2 \beta_2 t} - \frac{s_g \gamma_3 \bar{K}_g}{s_2 \beta_2} \quad \dots (4')$$

As in the case of Chapter 5, we take the initial values of  $Y_1$ ,  $Y_2$ ,  $Y_g$  (after redistribution) to be

$$\bar{Y}_1 = 34 \quad \bar{Y}_2 = 56 \quad \bar{Y}_g = 10$$

Therefore at  $t = 0$ ,

$$\bar{Y}_1 = z_1 = 0.8 \alpha_1 K_1 \quad (\text{see Appendix VIII})$$

$$\bar{Y}_g = a_g \bar{K}_g \quad \dots (5)$$

As in Appendix VIII, we shall use the following values of the parameters

$$\begin{array}{llll} a_1 = 0.3 & \alpha_1 = 0.18 & \alpha_2 = 0.12 & r_3 = 0.05 \\ \beta_2 = 0.28 & s_1 = 0.2 & s_2 = 0.1 & s_g = 0.1 \end{array}$$

Using these values, equations (5) yield

$$\bar{K}_1 = 236$$

$$\bar{K}_g = 40 \quad \dots (6)$$

Substituting for  $\bar{K}_1$  and  $\bar{K}_g$  in the second equation of (1) we get

$$56 = 70.8 - 34 + 0.28 \bar{K}_2 + 8 \quad \dots (7)$$

$$\text{i.e. } \bar{K}_2 = 40$$

Let the values of  $K_1$ ,  $K_2$ ,  $K_g$  at  $t = 25$  be denoted by  $K_1(T)$ ,  $K_2(T)$  and  $K_g(T)$

Then equations (4) gives

$$K_1(T) = s_1 Z_1 t + \bar{K}_1 = 406$$

$$K_g(T) = 40 \quad \dots (8)$$

Likewise equation (4) gives

$$\begin{aligned} K_2(T) &= (40 + 7.1)e^{0.7} - 7.1 \\ &= 87.7 \end{aligned}$$

At  $t = 25$ ,  $Y_1$  remains at 34, but from (1)

$$Y_2 = 122 - 34 + 24.6 + 8 = 120.6$$

As in Chapter IV (see Sections 4.2 and 4.8) let us take the population proportions of rich to the poor as 1 : 4.

Taking the former as the unit of population,

$$\text{at } t = 0 \quad N_1 = 1 \quad N_2 = 4 \quad \dots (9)$$

Let us consider the following rates of population increase  
(as in Appendix VIII)

$$\begin{aligned} r_1 &= 1.5 \text{ per cent per annum} \\ r_2 &= 1.8 \text{ per cent per annum} \quad \dots \quad (10) \end{aligned}$$

Then at  $t = 25$ ,  $N_1$  and  $N_2$  will be

$$N_1 = 1.45$$

$$N_2 = 6.27$$

Therefore the incomes of the rich and the poor per unit of population would change as follows.

	<u>Rich</u>	<u>Poor</u>
Income per unit of population at $t = 0$	34	14
Income per unit of population at $t = 25$	23.4	19.2

If the rate of increase of population of the poor is also taken as 1.5 per cent per annum, then the income per unit of population at  $t = 25$  works out to 20.7 instead of 19.2. It would thus appear that although the incomes per unit of population as between the rich and the poor would narrow down considerably, their roles would not be reversed over the time horizon of 25 years, and within the data framework used.

APPENDIX X

(To Chapter 7)

Explanatory Note No.3 to Chapter 7In Regime I,  $\lambda = 0$  and  $\mu = 1$ 

Therefore the income equation (7.2) read as

$$\begin{aligned}
 Y_1 &= \alpha_1 K_1 \\
 Y_2 &= \alpha_2 K_1 + \beta_2 K_2 + \gamma_2 K_g \\
 Y_g &= \gamma_3 K_g
 \end{aligned}
 \dots (1)$$

Differentiating with respect to time we get

$$\begin{aligned}
 \dot{Y}_1 &= \alpha_1 \dot{K}_1 \\
 \dot{Y}_2 &= \alpha_2 \dot{K}_1 + \beta_2 \dot{K}_2 + \gamma_2 \dot{K}_g \\
 \dot{Y}_g &= \gamma_3 \dot{K}_g
 \end{aligned}
 \dots (2)$$

The Capital accumulation equations are:

$$\begin{aligned}
 \dot{K}_1 &= s_1 \alpha_1 K_1 \\
 \dot{K}_2 &= s_2 \beta_2 K_2 + (s_g \gamma_3 K_g - I_g) \\
 \dot{K}_g &= I_g
 \end{aligned}
 \dots (3)$$

As in the case of Chapter 4 let us take  $\frac{Y_2}{Y_1}$  as

a measure of income inequality

$$\text{Then } \frac{d}{dt} \left( \frac{Y_2}{Y_1} \right) = \frac{Y_1 \dot{Y}_2 - Y_2 \dot{Y}_1}{Y_1^2}$$

From equations (1) and (2) we get

$$\begin{aligned}
 Y_1 \dot{Y}_2 - Y_2 \dot{Y}_1 &= \alpha_1 K_1 [\alpha_2 \dot{K}_1 + \beta_2 \dot{K}_2 + \gamma_2 \dot{K}_g] - \alpha_1 \dot{K}_1 [\alpha_2 K_2 + \beta_2 K_2 + \gamma_2 K_g] \\
 &= \alpha_1 \beta_2 [K_1 \dot{K}_2 - \dot{K}_1 K_2] + \alpha_1 \gamma_2 [K_1 \dot{K}_g - \dot{K}_1 K_g]
 \end{aligned}$$

Substituting for the  $\dot{K}$ s from equation (3) we get

$$\begin{aligned} Y_1 \dot{Y}_2 - Y_2 \dot{Y}_1 &= \alpha_1 \beta_2 \left[ K_1 \{ s_2 \beta_2 K_2 + s_g \gamma_3 K_g - I_g \} - s_1 \alpha_1 K_1 K_2 \right] \\ &\quad + \alpha_1 \gamma_2 K_1 (I_g - K_g s_1 \alpha_1) \\ &= \alpha_1 \beta_2 \left[ K_1 K_2 (s_2 \beta_2 - s_1 \alpha_1) + K_1 (s_g \gamma_3 K_g - I_g) \right] \\ &\quad + \alpha_1 \gamma_2 K_1 (I_g - K_g s_1 \alpha_1) \end{aligned}$$

According to the values of parameters in Appendix VIII

$(s_2 \beta_2 - s_1 \alpha_1)$  is negative.

Again  $I_g - K_g s_1 \alpha_1 < s_g \gamma_3 K_g - K_g s_1 \alpha_1 = K_g (s_g \gamma_3 - s_1 \alpha_1) < 0$

The only positive term in the above expression is  $(s_g \gamma_3 K_g - I_g)$ .

Therefore if the other two terms which are negative and sufficiently large in magnitude,  $Y_1 \dot{Y}_2 - Y_2 \dot{Y}_1 < 0$

That is,  $(Y_2/Y_1)$  will decline, which means that income inequality will increase.

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